1. Given $A = 494$ and $B = 247646$. 
   (a) Find the prime factorizations of $A$ and $B$ and use them to find $\gcd(A, B)$. 
   (b) Find $\gcd(A, B)$ using the Euclidean Algorithm. 

2. Use the Chinese Remainder Theorem to find the smallest and second smallest nonnegative solutions to the system:
   
   $x \equiv 2 \mod 3$
   $x \equiv 4 \mod 5$
   $x \equiv 12 \mod 13$

3. For each of $n = 15, 223, 3643, 70646$ find the exact value $p_n$ of the $n^{th}$ prime (however you want) and then approximate value $a_n$ of the $n^{th}$ prime (using the Prime Number Theorem Corollary). Calculate the percentage error 
   
   $\frac{100|p_n - a_n|}{p_n}$
   
   for each.

4. Find all incongruent solutions mod 160 to the linear system:
   
   $156x \equiv 144 \mod 160$

5. Find all primitive roots for $n = 13$ as follows: First find the smallest positive primitive root. Then use the Theorem from class which yields all the remaining ones. Final answers should be least nonnegative residues.

6. It’s a fact that $r = 7$ is a primitive root mod 11.
   (a) Use this to construct a table of indices for this primitive root.
   (b) Use the table of indices to solve the equation: $x^2 \equiv 4 \mod 11$. Your answer(s) should be mod 11.
   (c) Use the table of indices to solve the equation: $3^x \equiv 4 \mod 11$. Your answer(s) should be mod 10.

7. Calculate the following Jacobi symbols:
   (a) $(\frac{2145}{1073})$
   (b) $(\frac{2145}{51127})$
8. Suppose you intercept the following ciphertext from Alice to Bob: [15 pts]

\[ 1044 \ 7185 \ 2976 \ 12786 \ 726 \ 11510 \ 12371 \ 4559 \]

You know that Bob’s public key is \((e, n) = (1999, 14317)\). Bob thinks this is secure because he doesn’t believe that his \(n\) can be factored easily. Factor \(n = 14317\), find \(\phi(n)\), find \(d\) and then decrypt the message. Be clear about the steps you take.

9. Determine if each of the following sets is well-ordered. If a set is not well-ordered give evidence. If a set is well-ordered no evidence is required. [15 pts]

(a) \(\{0, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\}\)
(b) \(2\mathbb{Z}\)
(c) \(\{0\} \cup \left\{ \frac{n}{n+5} \bigg| n \in \mathbb{Z}^+ \right\}\)

10. Suppose \(p \geq 7\) is an unknown prime. Find all solutions to \(x^2 + 6 \equiv 5x \pmod{p}\). Note that your solutions will be mod \(p\). [15 pts]

11. Consider the inequality: [15 pts]

\[ 3^n < n! \]

(a) Find the smallest positive integer \(n_0\) for which this is true. Do this however you wish.
(b) Prove by induction that \(3^n < n!\) for all \(n \geq n_0\).

12. Suppose \(p\) is an odd prime such that there is some \(a\) so that \(a\) is a quadratic residue of \(p\) but \(2a\) is a quadratic non-residue of \(p\). Prove that \(p \equiv \pm 3 \pmod{8}\). [15 pts]

13. Prove that for \(a, b \in \mathbb{Z}\) and \(n \in \mathbb{Z}^+\) that if \(a^n \mid b^n\) then \(a \mid b\). [15 pts]

14. Prove that if \(a, b, c \in \mathbb{Z}\) with \(\gcd(a, b) = 1\) and \(c \mid (a + b)\) then \(\gcd(c, a) = \gcd(c, b) = 1\). [15 pts]