1. Given $A = 73610350$ and $B = 198375$. 
   (a) Find the prime factorizations of $A$ and $B$ and use them to find $\gcd(A, B)$.  
   (b) Find $\gcd(A, B)$ using the Euclidean Algorithm. 

2. Use the Chinese Remainder Theorem to find the smallest and second smallest nonnegative solutions to the system:  
   \[
   \begin{align*}
   x &\equiv 1 \mod 2 \\
   x &\equiv 1 \mod 3 \\
   x &\equiv 1 \mod 11
   \end{align*}
   \]

3. For each of $n = 10, 107, 1157, 13181$ find the exact value $p_n$ of the $n^{th}$ prime (however you want) and then approximate value $a_n$ of the $n^{th}$ prime (using the Prime Number Theorem Corollary). Calculate the percentage error  
   \[
   \frac{100|p_n - a_n|}{p_n}
   \]
   for each. 

4. Find all incongruent solutions mod 92 to the linear system:  
   \[
   32x \equiv 60 \mod 92
   \]

5. Find all primitive roots for $n = 11$ as follows: First find the smallest positive primitive root. Then use the Theorem from class which yields all the remaining ones. Final answers should be least nonnegative residues. 

6. It’s a fact that $r = 14$ is a primitive root mod 17.  
   (a) Use this to construct a table of indices for this primitive root. 
   (b) Use the table of indices to solve the equation: $x^{10} \equiv 15 \mod 17$. Your answer(s) should be mod 17.  
   (c) Use the table of indices to solve the equation: $9^x \equiv 15 \mod 17$. Your answer(s) should be mod 16. 

7. Calculate the following Jacobi symbols:  
   (a) $\left(\frac{1296}{1727}\right)$ 
   (b) $\left(\frac{1296}{30659}\right)$
8. Suppose you intercept the following ciphertext from Alice to Bob: 

\[ 7942 \ 16505 \ 15259 \ 4732 \ 18529 \ 13316 \]

You know that Bob’s public key is \((e,n) = (1039,19177)\). Bob thinks this is secure because he doesn’t believe that his \(n\) can be factored easily. Factor \(n = 19177\), find \(\phi(n)\), find \(d\) and then decrypt the message. Be clear about the steps you take.

9. Determine if each of the following sets is well-ordered. If a set is not well-ordered give evidence. If a set is well-ordered no evidence is required.

(a) \(\{0, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\}\)

(b) \(\{0\} \cup \left\{\frac{n+2}{n} \mid n \in \mathbb{Z}^+\right\}\)

(c) \([0, \infty) \cap 2\mathbb{Z}\)

10. Suppose \(p \geq 7\) is an unknown prime. Find all solutions to \(x^2 + 6 \equiv 5x \pmod{p}\). Note that your solutions will be \(\pmod{p}\).

11. Consider the inequality: \(3^n < n!\)

(a) Find the smallest positive integer \(n_0\) for which this is true. Do this however you wish.

(b) Prove by induction that \(3^n < n!\) for all \(n \geq n_0\).

12. Suppose \(p\) is an odd prime such that there is some \(a\) so that \(a\) is a quadratic residue of \(p\) but \(2a\) is a quadratic non-residue of \(p\). Prove that \(p \equiv \pm 3 \pmod{8}\).

13. Prove that for \(a,b \in \mathbb{Z}\) and \(n \in \mathbb{Z}^+\) that if \(a^n \mid b^n\) then \(a \mid b\).

14. Prove that if \(a,b,c \in \mathbb{Z}\) with \(\gcd(a,b) = 1\) and \(c \mid (a+b)\) then \(\gcd(c,a) = \gcd(c,b) = 1\).