1. State the following three definitions:
   (a) Define what it means for \( \{x_n\} \to x_0 \).
   (b) Define what it means for a set \( S \subseteq \mathbb{R} \) to be closed.
   (c) Define what it means for a function \( f : D \to \mathbb{R} \) to be uniformly continuous.

2. State the Intermediate Value Theorem. Pick one hypothesis, remove it, and give a counterexample showing the new statement is false.

3. The following is true for any convergent sequence \( \{x_n\} \to x_0 \):
   \[
   \text{If } x_0 > 0 \text{ then } \exists N \in \mathbb{N}, \forall n \geq N, x_n > 0.
   \]
   State the converse and give a counterexample showing that the converse is false.

4. Prove using \( \epsilon-N \) that:
   \[
   \left\{ 2 - \frac{1}{n} + \frac{3}{n^2} \right\} \to 2
   \]

5. Consider \( f : \mathbb{R} \to \mathbb{R} \) defined by
   \[
   f(x) = \begin{cases} 
   2x & \text{if } x \leq 5 \\
   0 & \text{if } x > 5 
   \end{cases}
   \]
   Prove using the sequence definition of continuity that \( f(x) \) is continuous at \( x = 3 \).

6. Suppose \( f : \mathbb{R} \to \mathbb{R} \) is continuous and \( x_0 \in \mathbb{R} \) with \( f(x_0) > 0 \). Show that there exists some \( \alpha > 0 \) such that \( f(x) > 0 \) for all \( x \in (x_0 - \alpha, x_0 + \alpha) \).

7. Suppose \( D \) is sequentially compact and \( f : D \to \mathbb{R} \) is continuous. Prove that \( f(D) \) is sequentially compact.

8. Suppose \( \{x_n\} \) is a bounded sequence which has the property that for all \( n \in \mathbb{N} \) there is some \( n_1 > n \) with \( x_{n_1} > x_n \) and some \( n_2 > n \) with \( x_{n_2} < x_n \). Prove that \( \{x_n\} \) does not converge.