Math 411 Exam 1 Fall 2013

Directions: All problems must be completed using methods taught in the course. Each problem is worth 10 points for a total of 70 which will then be converted to a percentage.

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1. Prove that $\mathbb{R}$ is connected.

2. Suppose $\bar{u} \in \mathbb{R}^n$ with $||\bar{u}|| < 1$. Show that if $\bar{v} \in \mathbb{R}^n$ and $||\bar{v} - \bar{u}|| < 1 - ||\bar{u}||$ then $||\bar{v}|| < 1$.

3. Prove from the definition that the sequence $\left\{2 + \frac{1}{k}, 1 - \frac{2}{k^2}\right\}$ converges to $(2, 1)$.

4. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous such that $f(x) \geq 0$ for $x \in \mathbb{Q}$. Prove that $f(x) \geq 0$ for all $x \in \mathbb{R}$.

5. Prove from the definition that the closed ball $S = \left\{(x, y) \mid x^2 + y^2 \leq 4\right\}$ is path connected. Make sure you show that your path is actually inside the set.

6. Prove from the definitions that if $A \subseteq \mathbb{R}^n$ is open in $\mathbb{R}^n$ then $\mathbb{R}^n - A$ is closed in $\mathbb{R}^n$.

7. Give an example of each of the following. No justification is required. Pictures suffice if you wish.

   (a) A collection of open subsets of $\mathbb{R}$ whose intersection is not open.

   (b) Two pathwise connected sets whose intersection is not pathwise connected.

   (c) A non-convex pathwise connected set.