Math 411 Exam 2 Fall 2013

1. Find the first-order approximation for $f(x,y) = x^3 + xy$ at $(1,2)$ and use it to approximate the value of $f(1.1, 2.2)$. Simplify only to eliminate inner product calculations. [15pts]

2. Give an example of an invertible nonlinear transformation from $\mathbb{R}^2 \to \mathbb{R}^2$. Prove that it is invertible and prove that it is nonlinear. [15pts]

3. Let m and n be positive integers with $m + n > 2$. Show that the following limit exists: [10pts]

$$\lim_{(x,y)\to(0,0)} \frac{x^n y^m}{x^2 + y^2}$$

4. Define $f(x,y) = xy + x^2$. Let $\bar{x} = (0,0)$ and $\bar{h} = (2,1)$. Find the specific value of θ that satisfies the condition of the Mean Value Theorem. [15pts]

5. Suppose that $f : \mathbb{R}^2 \to \mathbb{R}$ is continuously differentiable and $\nabla f(0,0) \neq (0,0)0$. Show that there exists a vector $\bar{p} \neq \bar{0}$ such that $\frac{\partial f}{\partial \bar{p}}(0,0) > 0$. [10pts]

6. Let $a > 0$ be unknown and define $f(x,y) = x^2 + axy + ay^2$. First check that $(0,0)$ is the only critical point for f and then find the conditions on a that would make $(0,0)$ a relative minimum according to the Second Derivative Test. [20pts]

7. Suppose $\bar{F} : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $F(x,y) = (\phi(x,y), \psi(x,y))$ and $g : \mathbb{R}^2 \to \mathbb{R}$.

   (a) Calculate $\nabla(g \circ \bar{F})(x,y)$ as far as possible using the chain rule. [10pts]

   (b) Assuming $g(u,v) = u^2 v^3$, calculate further. [5pts]