1. Find the first-order approximation for \( \bar{F}(x, y) = (x^2 y, xy + y) \) at \((-1, 2)\) and use it to approximate the value of \( \bar{F}(-0.9, 2.1) \).

2. (a) Given a transformation \( T: \mathbb{R}^n \to \mathbb{R}^m \), what is logically incorrect about finding the matrix 
\[
[T(\bar{e}_1) \ldots T(\bar{e}_n)]
\]
and then using this matrix to show that the transformation is linear?

(b) Let \( A \) be the set of all linear transformations and \( B \) be the set of all invertible transformations. Give two transformations, one which proves that \( A \subsetneq B \) and one which proves that \( B \subsetneq A \).

3. Let \( f(x, y) = xy + y^2 \) and \( \bar{p} = (-1, 2) \). Find \( \frac{\partial f}{\partial \bar{p}}(1, 1) \) using the limit definition of the directional derivative and also using the inner product calculation.

4. Suppose \( \bar{F}(x, y) = (x^2 y, y - 3x^2) \) and \( \bar{G}(x, y) = (xy + y, y - xy) \). Use the matrix form of the chain rule to evaluate \( D(\bar{F} \circ \bar{G})(x, y) \).

5. Define \( f(x, y) = 2x^2 - 2xy - y^2 \). Find the only critical point \((x_0, y_0)\) and show that the Hessian at \((x_0, y_0)\) is neither positive definite nor negative definite. Moreover show that there is at least one direction \( \bar{h}_1 \) in which \( \langle \nabla^2 f(x_0, y_0) \bar{h}_1, \bar{h}_1 \rangle > 0 \) and another direction \( \bar{h}_2 \) in which \( \langle \nabla^2 f(x_0, y_0) \bar{h}_2, \bar{h}_2 \rangle < 0 \).

6. Define
\[
f(x, y) = \begin{cases} 
x \sqrt{x^2 + y^2} & \text{if } y \neq 0 \\
0 & \text{if } y = 0
\end{cases}
\]
Show that \( f \) has directional derivatives in all directions at \((0, 0)\).

7. Suppose \( f: \mathbb{R}^2 \to \mathbb{R} \) is continuously differentiable with \( f(0, 0) = 1 \) and \( f(x, y) = 1 \) for all \( ||(x, y)|| = 1 \). Show that there is some point \((x_0, y_0)\) such that \( \frac{\partial f}{\partial x}(x_0, y_0) = \frac{\partial f}{\partial y}(x_0, y_0) \).
Hint: Use the MVT for an appropriate \( \bar{h} \).