MATH 411 (JWG) Exam 2 Spring 2021

Due by Thu Apr 15 at 12:00pm

Exam Logistics:

1. From the moment you download this exam you have three hours to take the exam and submit to Gradescope. This includes the entire upload and tag procedure so do not wait until the last minute to do these things.
2. Tag your problems! Please! Pretty please!
3. You may print the exam, write on it, scan and upload.
4. Or you may just write on it on a tablet and upload.
5. Or you are welcome to write the answers on separate pieces of paper if other options don't appeal to you, then scan and upload.

Exam Rules:

1. You may ask for clarification on questions but you may not ask for help on questions!
2. You are permitted to use official class resources which means your own written notes, class Panopto recordings and the textbook.
3. You are not permitted to use other resources. Thus no friends, internet, calculators, Wolfram Alpha, etc.
4. By taking this exam you agree that if you are found in violation of these rules that the minimum penalty will be a grade of 0 on this exam.

Exam Work:

1. Show all work as appropriate for and using techniques learned in this course.
2. Any pictures, work and scribbles which are legible and relevant will be considered for partial credit.
3. Arithmetic calculations do not need to be simplified unless specified.

1. Define the function $f : \mathbb{R}^2 \to \mathbb{R}$ by:

$$f(x,y) = \begin{cases} \frac{2x^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 2 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) Prove using the limit definition that f only has one partial derivative at $(0,0)$. [10 pts]

Solution:

(b) Which value would need to replace 2 so that the other partial derivative exists instead? Explain. [5 pts]

Solution:

2. If $f(x, y)$ gives the temperature in celsius of the plane at the point (x, y) in meters and if $\bar{p} = (1, 1)$ explain in your own words the meaning of the following expression. [10 pts]

$$\left\langle \nabla \left\langle \nabla f(x, y), \bar{e}_i \right\rangle, \bar{p} \right\rangle$$

Solution:

3. Let $f(x, y) = xy + x$ and $\bar{p} = (-1, 2)$.

   (a) Show that the limit definition of $\frac{\partial f}{\partial \bar{p}}(\bar{x})$ and the convenient $\langle \nabla f(\bar{x}), \bar{p} \rangle$ yield the same result for any $\bar{x}$. [10 pts]

   Solution:

   (b) Given $\bar{x} = (1, -3)$, find the specific value of θ satisfying the conditions of the Mean Value Theorem. [10 pts]

   Solution:

4. Given the function $\bar{F} : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $\bar{F}(x, y, z) = (x^2y + yz^2, xyz)$.

(a) Find the first-order approximation (in the $\bar{F}(x, y, z) \approx ...$ form) of $\bar{F}$ at the point $(x_0, y_0, z_0) = (1, 2, 3)$. You can leave this as a matrix expression. [10 pts]

Solution:

(b) Use your answer to (a) to approximate $\bar{F}(1.1, 1.9, 3.01)$. This should be simplified. [5 pts]

Solution:

5. Given the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = x^3y - xy^3 + x^2y^2$.

   (a) Find the second-order approximation (in the $f(x, y) \approx ...$ form) of f at the point $(x_0, y_0) = (2, 3)$. You can leave this as a matrix expression. [10 pts]

   Solution:

   (b) Is the Hessian matrix at $(2, 3)$ positive definite, negative definite, or neither? Justify. [10 pts]

   Solution:

6. Supppose $\phi : \mathbb{R}^2 \to \mathbb{R}$ is continuously differentiable and define $f(x, y) = \phi(x+y, x-y)$. Express $\nabla f(x, y)$ as a vector involving combinations of $\frac{\partial \phi}{\partial x_1}(x + y, x - y)$ and $\frac{\partial \phi}{\partial x_2}(x + y, x - y)$. [10 pts]

Solution: