1. Prove that the subset of \( \mathbb{R} \) defined by \( \{0\} \cup \left\{ \frac{1}{n} \mid n \in \mathbb{N} \right\} \) has Jordan content zero.

2. Define \( R = [0,1] \times [0,1] \) and \( f : R \to \mathbb{R} \) as follows.

If \( x \in \mathbb{Q} \) then put \( f(x,y) = \begin{cases} 1 & \text{if } 0 \leq y \leq \frac{1}{2} \\ 0 & \text{if } \frac{1}{2} < y \leq 1 \end{cases} \)

If \( x \notin \mathbb{Q} \) then put \( f(x,y) = \begin{cases} 0 & \text{if } 0 \leq y \leq \frac{1}{2} \\ 1 & \text{if } \frac{1}{2} < y \leq 1 \end{cases} \).

Find \( \int_0^1 \int_0^1 f(x,y) \, dy \, dx \).

3. Define \( S \subseteq \mathbb{R}^2 \) by \( S = \{(x,y) \mid x - y^2 + 6y = 0\} \).

   (a) Find all points \((x_0,y_0)\) where Dini’s Theorem shows that \( S \) is locally a function of \( x \) and find the slope of that local function at \( x_0 \).

   (b) Show that at every point not satisfying the hypotheses of Dini’s Theorem \( S \) is not locally a function of \( x \).

4. Suppose the temperature and humidity \((t,h)\) of a storage container is set by two controls \((x,y)\) such that \( t = x^2 + 5y \) and \( h = xy + y^3 \). You set the controls at \((x,y) = (2,3)\) to achieve the desired \((t,h) = (27,33)\). Show that for any \((t,h)\) sufficiently close to \((27,33)\) you can alter \((x,y)\) slightly in order to achieve this new \((t,h)\). Moreover, approximately (linearly) which \((x,y)\) would you set in order to achieve \((t,h) = (28,34)\)?

5. Define \( D = ([0,1] \cap \mathbb{Q}) \times ([0,1] \cap \mathbb{Q}) \times ([0,1] \cap \mathbb{Q}) \). Show that the volume of \( D \) is undefined.

6. Define the change of variables \( \Psi \) by \( \Psi(u,v) = \left( \frac{u}{u+v}, \frac{u}{u+v} \right) \) for \( u,v > 0 \).

   Define \( D = \left\{ (x,y) \mid 1 \leq x + y \leq 4 \text{ and } \frac{1}{2} \leq \frac{y}{x} \leq 3 \right\} \). Use \( \Psi \) to rewrite \( \int_D \left( y + \frac{y^2}{x} \right) \) as an iterated integral in \( u \) and \( v \). Justify

7. Define \( D = [0,1] \times [0,1] \) and \( f : D \to \mathbb{R} \) by \( f(x,y) = x \). Using lower and upper sums show that \( f \) is integrable over \( D \) and find the integral.