1. Write down the matrix product in $\mathbb{RP}^2$ which rotates the four points $[1 : 2], [0 : 0], [-5 : 2]$ and $[8 : 10]$ by 4.1 radians about the point $(56, 70)$.

2. Suppose a transformation of the plane is expressed as $z \mapsto e^{i\pi/6}(z - (2 + 4i))$ when treating the plane as $\mathbb{C}$. Rewrite this transformation as a product of matrices in $\mathbb{RP}^2$.

3. Suppose the point and normal pair $(v_0, n_0)$ represents a plane $\mathcal{P}$ and the point and direction pair $(v_1, d_1)$ represents a line $\mathcal{L}$.
   
   (a) If $\mathcal{P}$ is rotated by $\theta$ radians about $\mathcal{L}$ according to the right-hand rule applied to $\mathcal{L}$, write down the algebraic quaternion expressions (point and normal pair) for the resulting plane.

   (b) If $\mathcal{L}$ is reflected in $\mathcal{P}$, write down the algebraic quaternion expressions (point and direction pair) for the resulting line.
4. In $\mathbb{RP}^2$ if you follow the ends of the parabola $x = y^2$, which point(s) at infinity do they approach?

5. Suppose $L$ is a line in $\mathbb{C}$ represented by the closest point to the origin $z_0 \in \mathbb{C}$. Suppose $L$ is rotated about the point $c_0 \in \mathbb{C}$. Describe a criteria (with words, pictures, or algebraically) under which the rotation of $L$ would never pass through the origin, no matter what angle of rotation was used.

6. In $\mathbb{C}$ show that the composition of two rotations by different angles about different points can be written as a single rotation followed by a translation.
7. Suppose the location of an object is given by the parametrization \( \mathbf{r}(t) = t \mathbf{i} - t^2 \mathbf{j} \) for \( t \geq 0 \). Consider the perspective projection with \( y = 10 \).

(a) Find the projection of the object on the \( x \)-axis at time \( t \).

(b) Where does the projection of the object go as \( t \to \infty \)?

(c) How far to the right on the \( x \)-axis does the object go before reversing course?
8. Show algebraically using $\mathbb{H}$ that rotation by $\theta$ followed by rotation by $-\theta$ cancel. [Ch5:10pts]

9. Suppose $\mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ are perpendicular. Write down an expression using simple vector operations (no quaternions) for the result of rotating $\mathbf{v}$ by $\theta$ radians in the direction of $\mathbf{w}$. [Ch2:10pts]