MATH 431 Homework 5 Solutions

(6.2) Which projective points in \( E^2 \subset \mathbb{RP}^2 \) correspond to each of the following points in \( \mathbb{R}^2 \).

(a) \( [2; 1] \)
   Solution: \( [2; 1; 1] \)

(b) \( [1; 6] \)
   Solution: \( [1; 6; 1] \)

(c) \( [0.1; 0.7] \)
   Solution: \( [0.1; 0.7; 1] \)

(6.3) Which points in \( \mathbb{R}^2 \) correspond to each of the following points in \( \mathbb{RP}^2 \). One is a trick.

(a) \( [2; 5; -2] \)
   Solution: \( [-1; -5/2] \)

(b) \( [10; 5; 7] \)
   Solution: \( [10/7; 5/7] \)

(c) \( [7; 2; 0] \)
   Solution: None

(d) \( [6; 0.2; 0.15] \)
   Solution: \( [6/0.15; 0.2/0.15] \)

(6.6) Write down the (approximate) matrix which rotates counterclockwise by 4.12 radians about the point \([5; 2]\) and show how it rotates the point \([10; 10]\).

Solution: The matrix is:

\[
\begin{bmatrix}
-0.5583 & 0.8296 & 6.1325 \\
-0.8296 & -0.5583 & 7.2647 \\
0 & 0 & 1.0000
\end{bmatrix}
\]

and:

\[
\begin{bmatrix}
-0.5583 & 0.8296 & 6.1325 \\
-0.8296 & -0.5583 & 7.2647 \\
0 & 0 & 1.0000
\end{bmatrix}
\begin{bmatrix}
10 \\
10 \\
1
\end{bmatrix}
= 
\begin{bmatrix}
8.8452 \\
-6.6148 \\
1.0000
\end{bmatrix}
\]

(6.8) If your eye is at \([0; 10]\) calculate where the point \([8; -5]\) gets mapped on the \(x\)-axis by this perspective projection.

Solution: We have:

\[
\begin{bmatrix}
1.0 & 0 & 0 \\
0 & 0 & 0 \\
0 & -0.1 & 1.0
\end{bmatrix}
\begin{bmatrix}
8 \\
-5 \\
1
\end{bmatrix}
= 
\begin{bmatrix}
8.00 \\
0 \\
1.50
\end{bmatrix}
\equiv 
\begin{bmatrix}
5.33 \\
0 \\
1.00
\end{bmatrix}
\]
(6.11) For the line defined by \([5; 2; 6]\) which Euclidean equation does this correspond to in \(E^2 \equiv \mathbb{R}^2\) and which point at infinity is on this line?

**Solution:** This corresponds to the line \(5x + 2y + 6 = 0\) and the point at infinity is \([-b; a; 0] = [-2; 5; 0]\). Any nonzero multiple of this is acceptable too.

(6.12) Which point at infinity is picked up when the line \(y = 5x - 7\) is moved into \(\mathbb{RP}^2\)?

**Solution:** This line is \(5x - y - 7 = 0\) and is represented by \([5; -1; -7]\) so the point at infinity is \([-b; a; 0] = [1; 5; 0]\). Any nonzero multiple of this is acceptable too.

(6.15) Use the above method to calculate the point at which the lines \(y = 5x + 1\) and \(y = -2x - 7\) meet.

**Solution:** We set \(L_1 = [5; -1; 1]\) and \(L_2 = [-2; -1; -7]\) or any nonzero multiples of these and then: \(L_1 \times L_2 = [-6; -37; -7] \equiv [6/7; 37/7; 1]\) so they meet at \([6/7; 37/7; 1]\).

(6.19) Use the above method to calculate the Euclidean equation of the line joining the points \([2; 3]\) and \([-4; 9]\).

**Solution:** We set \(P_1 = [2; 3; 1]\) and \(P_2 = [-4; 9; 1]\) or any nonzero multiples of these and then: \(P_1 \times P_2 = [-6; -6; 30]\) which represents the line \(-6x - 6y + 30 = 0\) or \(x + y - 5 = 0\).

(6.20) Check if the following triples are colinear.

(a) \([2; 2; 2], [5; 13; 1], \) and \([-6; -24; -3]\)

**Solution:** Since \(((2; 2; 2) \times [5; 13; 1]) \cdot [-6; -24; -3] = -96 \neq 0\) they are not. This can be done in any order provided you cross two and dot with the third.

(b) \([5; -1; 1], [16; -8; 2]\) and \([2; 2; 0]\)

**Solution:** Since \(((5; -1; 1) \times [16; -8; 2]) \cdot [2; 2; 0] = 24 \neq 0\) they are not.

(6.22) Suppose \(V_1, V_2, V_3, V_4 \in \mathbb{RP}^2\). Consider the calculation \((V_1 \times V_2) \times (V_3 \times V_4)\) What does this calculation do in the following cases:

(a) If the \(V_i\) represent lines?

**Solution:** If they represent lines then the calculation finds the vector which represents the line joining the intersection of the pairs \(V_1, V_2\) and \(V_3, V_4\).

(b) If the \(V_i\) represent points?

**Solution:** If they represent points then the calculation finds the point of intersection of the two lines, one which connects \(V_1, V_2\) and the other which connects \(V_3, V_4\).

(6.26) Suppose \(L = [5; 3; 1]\) represents a line in \(\mathbb{RP}^2\). If we rotate this line by \(R_\gamma(\pi/4)\) which vector represents the resulting line?

**Solution:** The vector is:

\[
(R_\gamma(\pi/4)^{-1})^T \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 1 \end{bmatrix}
\]