Math 341 FINAL EXAM Spring 2010 M. Brin

Each problem is 30 points. Show all work. Justify and simplify all answers.
Solve each problem on a separate answer sheet.

1. Solve explicitly (find y as a function of x) the initial value problem

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, \quad y(0) = -1.$$

2. Suppose that a room, containing 1200 ft^3 of air, is originally free of CO (carbon monoxide). Beginning at time $t = 0$ cigarette smoke, containing 4% CO, is introduced into the room at a rate of 0.1 ft^3/min, and the well-circulated mixture leaves the room at the same rate through a window. Find the concentration $x(t)$ of CO in the room at any time $t > 0$.

3. Solve (in real form): $y'' + 4y' + 5y = 0, \quad y(0) = 1, \ y'(0) = 0.$

4. Find the general solution of $y'' - 2y' + y = xe^x$.

5. For the vibrating system $\ddot{x} + 2\dot{x} + 2x = \sin 2t$ find the steady state (in real form) and its amplitude.

6. Find the general solution of

a) $\dot{\mathbf{x}} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \mathbf{x}$ b) $\dot{\mathbf{x}} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ e^t \end{pmatrix}$.

7. For the system $\dot{x} = 1 - y$, $\dot{y} = x^2 - y^2$,

a) find all critical points,

b) determine their type and stability,

c) sketch a phase portrait for the linearized system near each critical point.