Moy-Prasad filtrations and harmonic analysis: an example

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1 Introduction

One of the interesting and relatively recent developments in the character theory of reductive p-adic groups is the work of Fiona Murnaghan involving the relationship between supercuspidal characters and Fourier transforms of elliptic orbital integrals. In this note, I show how the formalism of Moy and Prasad can be used to refine this relationship. More details can be found in [3].

The results discussed below owe their existence, in part, to conversations I have had with many people, including Jeff Adler, Fiona Murnaghan, Allen Moy, Gopal Prasad, Robert Kottwitz, and my adviser Professor Sally. In particular, I thank Fiona Murnaghan for sharing with me her proof of the result at the end of this paper in the case where our representation is an unramified irreducible supercuspidal representation as constructed in [2].

2 Notation

We will need a fair amount of notation.

2.1 General notation

Let F denote a nonarchimedian local field. Let R be its ring of integers and $\wp = \varpi R$ the prime ideal. We fix an additive character, Λ, with conductor $\wp$.

We will always take our group G to be $\mathrm{GL}_n(F)$ realized as the set of $n \times n$ matrices with nonzero determinant. We denote by $\mathfrak{g}$ the Lie algebra of G. We will take $\mathfrak{g}$ to be $M_n(F)$ with the usual bracket operation. $\mathcal{N}$ denotes the set of nilpotent elements in $\mathfrak{g}$, i.e., the set

$$\{X \in \mathfrak{g} \colon X^n = 0\}.$$

Regular will always mean regular semisimple and G^{reg} will denote the set of regular elements in G. (Similar notation applies to $\mathfrak{g}$.)

For $g \in G$ and $X \in \mathfrak{g}$, we denote by ${}^gX$ the adjoint action of g on X. If S is a subset of $\mathfrak{g}$, then

$${}^GS = \{{}^gs \colon s \in S, g \in G\}$$

denotes the G-orbit of S.

2.2 Some filtrations for $\mathrm{GL}_n(F)$

2.2.1 Congruence filtration

The congruence filtration lattices are:

$$\mathfrak{k}_0 = M_n(R)$$

$$\mathfrak{k}_i = \varpi^i \cdot \mathfrak{k}_0$$

The corresponding congruence filtration subgroups are:

$$K_0 = \mathfrak{k}_0^\times$$

$$K_i = 1 + \mathfrak{k}_i$$

for $i > 0$.

2.2.2 Standard Iwahori filtration

The standard Iwahori filtration lattices in $\mathfrak{g}$: Let $\mathfrak{b}_0$ denote the inverse image under the "reduction mod $\wp$" map $\mathfrak{k}_0 \longrightarrow M_n(\mathbb{F}_q)$ of the standard Borel subalgebra in $M_n(\mathbb{F}_q)$. I.e.,

$$\mathfrak{b}_0 = \left\{ X \in \mathfrak{k}_0 \,\middle|\, X_{ij} \in \wp \text{ if } i > j \right\}.$$

Define

$$\Pi = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 \\ \varpi & 0 & 0 & 0 & \dots & 0 & 0 & 0 \end{bmatrix}$$

and let

$$\mathfrak{b}_i = \Pi^i \cdot \mathfrak{b}_0.$$

The corresponding Iwahori filtration subgroups are:

$$B_0 = \mathfrak{b}_0^\times$$

$$B_i = 1 + \mathfrak{b}_i$$

for $i > 0$.

Example 1. If $n = 3$, then

$$\mathfrak{b}_0 = \begin{pmatrix} R & R & R \\ \wp & R & R \\ \wp & \wp & R \end{pmatrix} \supset \mathfrak{b}_1 = \begin{pmatrix} \wp & R & R \\ \wp & \wp & R \\ \wp & \wp & \wp \end{pmatrix} \supset \mathfrak{b}_2 = \begin{pmatrix} \wp & \wp & R \\ \wp & \wp & \wp \\ \wp^2 & \wp & \wp \end{pmatrix} \supset \mathfrak{b}_3 = \varpi \cdot \mathfrak{b}_0.$$

2.3 Fourier transform

For $f \in C_c^\infty(\mathfrak{g})$, we define the Fourier transform of f by

$$\widehat{f}(X) = \int_{\mathfrak{g}} dY\, f(Y) \cdot \Lambda\big(\mathrm{tr}(X \cdot Y)\big)$$

for $X \in \mathfrak{g}$. Note that $\widehat{f}$ is again an element of $C_c^\infty(\mathfrak{g})$. In fact,

$$f \in C(\mathfrak{k}_i/\mathfrak{b}_j) \text{ if and only if } \widehat{f} \in C(\mathfrak{b}_{1-j}/\mathfrak{k}_{1-i}).$$

2.4 Moy-Prasad filtrations on $\mathfrak{g}$

Moy and Prasad in [8, 9] have defined a way to associate a lattice $\mathfrak{g}_{x,r}$ in $\mathfrak{g}$ to a pair (x, r) in $\mathcal{B}(G) \times \mathbb{R}$ (the product of the Bruhat-Tits building of G and $\mathbb{R}$). These lattices have the property that for $g \in G$,

$${}^g\mathfrak{g}_{x,r} = \mathfrak{g}_{gx,r}.$$

Fix a chamber C in $\mathcal{B}(G)$. We know that any point in $\mathcal{B}(G)$ is conjugate to a point in $\bar{C}$, the closure of C. Therefore, in order to understand the lattices $\mathfrak{g}_{x,r}$ up to conjugation, it is enough to understand the $\mathfrak{g}_{x,r}$ for $(x, r) \in \bar{C} \times \mathbb{R}$.

Rather than repeat the definition of $\mathfrak{g}_{x,r}$, we offer a few illustrations in the case where $G = \mathrm{SL}_2(F)$. Let us choose the apartment, $\mathcal{A}$, in $\mathcal{B} = \mathcal{B}(\mathrm{SL}_2(F))$ corresponding to the diagonal torus in $\mathrm{SL}_2(F)$. Let $\mathcal{I}$ be the set of matrices of the form

$$\begin{pmatrix} R & R \\ \wp & R \end{pmatrix}$$

in $\mathrm{SL}_2(F)$. Let C be the chamber in $\mathcal{A}$ with stabilizer $\mathcal{I}$. In Figure 1, we illustrate how the lattice $\mathfrak{g}_{x,r}$ varies as x and r vary for x near C (C is the open line segment with endpoints λ_0 and $\lambda_1 \otimes \frac{1}{2}$). Since the apartment $\mathcal{A}$ can be thought of as a copy of $\mathbb{R}$, this figure may be thought of as a picture of $\mathbb{R}^2$. Each diagonal dotted line is the graph of $r = \psi(x)$ for some affine root ψ. The horizontal dotted lines correspond to the natural filtration of the diagonal Cartan subalgebra in $\mathfrak{sl}_2(F)$.

The dotted lines divide the plane into convex polygons. Fix one such polygon D. If (x_1, r_1) and (x_2, r_2) are two points in the interior of D, then $\mathfrak{g}_{x_1,r_1} = \mathfrak{g}_{x_2,r_2}$. Because of the conditions which define $\mathfrak{g}_{x,r}$ for arbitrary x and r, a point (y, s) on the boundary of D corresponds to the same filtration lattice as every other point in the interior of D if and only if there exists a point (y, r) in the interior of D with $r < s$.

Unfortunately, the Moy-Prasad filtration lattices which occur for $\mathfrak{sl}_2$ are all familiar. Less familiar lattices do occur in, for example, $\mathfrak{sl}_3$.

2.5 Some comments on the Moy-Prasad filtrations

As a notational convenience, we define $\mathfrak{g}_{x,r^+} = \cup_{s>r}\mathfrak{g}_{x,s}$. Note that for the example in §2.4, we have $\mathfrak{g}_{x,r} = \mathfrak{g}_{x,r^+}$ unless (x, r) lies on a dotted line.

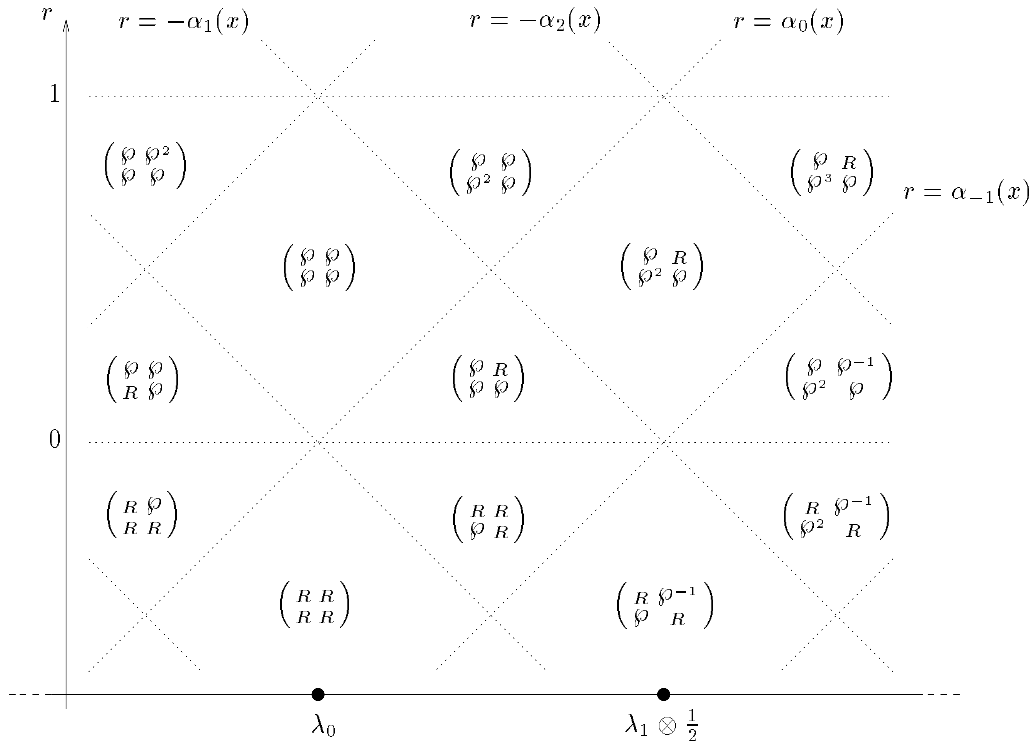

Figure 1: $(\mathfrak{sl}_2)_{x,r}$, x near C

For $\mathrm{GL}_n(F)$ the Fourier transform behaves well with respect to the Moy-Prasad filtrations (for other groups, the Fourier transform behaves well if you realize the Fourier transform as a function on the dual space of $\mathfrak{g}$). In particular, we have

$$f \in C(\mathfrak{g}_{x,r}/\mathfrak{g}_{y,s}) \text{ if and only if } \hat{f} \in C(\mathfrak{g}_{y,(-s)^+}/\mathfrak{g}_{x,(-r)^+}).$$

Finally, if we restrict r to the nonnegative real numbers, then we can define filtration subgroups, $G_{x,r}$, of the parahoric subgroup G_x attached to x.

3 Moy-Prasad filtrations and harmonic analysis

Now that we have completed our notational tour, we turn to a result.

Theorem 2 (DeBacker).

1.

$$\bigcup_{x \in \mathcal{B}(G)} \mathfrak{g}_{x,r} = \bigcap_{x \in \mathcal{B}(G)} (\mathfrak{g}_{x,r} + \mathcal{N})$$

2. If $P = MN$ is a parabolic subgroup of G with Lie algebra $\mathfrak{p} = \mathfrak{m} + \mathfrak{n}$, then

$$\Big(\bigcup_{x \in \mathcal{B}(G)} \mathfrak{g}_{x,r}\Big) \cap \mathfrak{m} = \bigcup_{x \in \mathcal{B}(M)} \mathfrak{m}_{x,r}$$

This theorem is true for any reductive p-adic group G defined over F. Since we have not defined the $\mathfrak{g}_{x,r}$, we cannot prove this theorem here. So I will offer some commentary instead.

3.1 Asymptotics

In [6, Lemma 2.4] Howe shows that

$$^G\mathfrak{k}_i \subset \mathfrak{k}_i + \mathcal{N}. \qquad (*)$$

The first statement of Theorem 2 allows a generalization of this result. For any x and y in $\mathcal{B}(G)$, we have $\mathfrak{g}_{x,r} \subset \mathfrak{g}_{y,r} + \mathcal{N}$. Or, to make it look like $(*)$:

$$^G\mathfrak{g}_{x,r} \subset \mathfrak{g}_{y,r} + \mathcal{N}.$$

By choosing appropriate x, y, and r we can say, for example:

$$^G\mathfrak{b}_i \subset \mathfrak{b}_i + \mathcal{N},$$

$$^G\mathfrak{b}_1 \subset \mathfrak{k}_1 + \mathcal{N},$$

or

$$^G\mathfrak{k}_i \subset \varpi^i\mathfrak{b}_0 + \mathcal{N}.$$

This final asymptotic result is a strengthening of $(*)$, and we will make use of it later.

3.2 G-domains

Since orbital integrals, characters, etc. are G-invariant functions, we need G-invariant sets with nice properties in order to properly analyze these functions. Since a G-invariant set cannot be compact, the most we can ask for is that it be closed. We therefore make the following definition: a *G-domain* is a closed, open, G-invariant subset of $\mathfrak{g}$.

Definition 3.

$$V_r = \bigcup_{x \in \mathcal{B}(G)} \mathfrak{g}_{x,r}$$

V_r is clearly G-invariant and open. The first statement in Theorem 2 shows that V_r is also closed. Thus, V_r is a G-domain. In fact, the V_r form a neighborhood basis of $\mathcal{N}$.

Example 4.

1. $V_0 = {}^G\mathfrak{k}_0$. V_0 is sometimes called the set of compact elements in $\mathfrak{g}$.

2. $V_{1/n} = V_{0^+} = {}^G\mathfrak{b}_1$. V_{0^+} is often called the set of topologically nilpotent elements in $\mathfrak{g}$.

3. Recall the example of $\mathrm{SL}_2(F)$ in §2.4. If $G = \mathrm{SL}_2(F)$, we have

$$V_0 = {}^G\begin{pmatrix} R & R \\ R & R \end{pmatrix} \cup {}^G\begin{pmatrix} R & \wp^{-1} \\ \wp & R \end{pmatrix}$$

and

$$V_{1/2} = {}^G\begin{pmatrix} \wp & R \\ \wp & \wp \end{pmatrix}.$$

Note that up to scaling, these are the only two G-domains of the form V_r which occur.

We can assign a unique rational number to every element of $X \in \mathfrak{g} \setminus \mathcal{N}$. We call this number $m(X)$, the level of X. The level is assigned to X by the rule: $X \in V_{m(X)} \setminus V_{m(X)^+}$. Allen Moy has proved a similar result. The notion of "level" is very convenient when writing down character tables. In particular, it allows one to avoid the traditional method of describing the character on each conjugacy class of maximal tori.

We will make use of the V_r when working out the example at the end of this note.

3.3 Parabolic descent

The idea for proving the second statement of Theorem 2 came from the second paper of Moy and Prasad where they show that parabolic induction preserves depth.

The second statement of Theorem 2 allows one to make precise statements about where the local expansion of a parabolically induced distribution on $\mathfrak{g}$ must hold if you know something about where the local expansion is valid for the distribution being induced. That is, suppose that T is an M-invariant distribution on $\mathfrak{m}$ with a local expansion on $\cup_{x \in \mathcal{B}(M)} \mathfrak{m}_{x,r}$. Then $i_P^G T$ has a local expansion on V_r.

It also allows one to show that proving Waldspurger-type uniformity results about distributions on $\mathfrak{g}$ (see [15]) is equivalent to proving statements about particular elliptic orbital integrals.

4 An example

We need to cover some more material before we can look at an example.

4.1 Background

Suppose that (π, W) is an admissible, irreducible representation of G. Then the character distribution

$$\Theta_\pi : C_c^\infty(G) \to \mathbb{C}$$

which sends f to $\operatorname{tr}\bigl(\pi(f)\bigr)$ is represented by a locally integrable function on G which is locally constant on G^{reg}. We abuse notation and denote this function by Θ_π. That is, if f is a locally constant, compactly supported function on G, then we have

$$\Theta_\pi(f) = \int_G dg\, f(g) \cdot \Theta_\pi(g)$$

where dg is a Haar measure on G. This was proved in [5] in characteristic zero and in [7] for $\mathrm{GL}_n(F)$ with F having arbitrary characteristic.

Moy and Prasad define the *depth*, $\rho(\pi)$, of π. A property of $\rho(\pi)$ is that it is the smallest number for which there exists an $x \in \mathcal{B}(G)$ such that

$$W^{G_{x,\rho(\pi)}} = \{0\}$$

while

$$W^{G_{x,\rho(\pi)+}} \neq \{0\}.$$

Suppose that $X \in \mathfrak{g}^{\text{reg}}$ is elliptic (X is elliptic if every eigenvalue of X generates a degree n extension of F). We have the distribution

$$\widehat{\mu_X} : C_c^\infty(\mathfrak{g}) \to \mathbb{C}$$

which maps f to

$$\int_{G/Z} dg^*\, \widehat{f}({}^g X).$$

$\widehat{\mu_X}$ is represented by a locally integral function on $\mathfrak{g}^{\text{reg}}$ which is locally constant. This is proved in characteristic zero in [5], and extending it to positive characteristic is not a problem for $\mathrm{GL}_n(F)$.

4.2 The example

Theorem 5 (Example). *Suppose that (π, W) is an irreducible supercuspidal representation as constructed in [2]. Then there exists a regular elliptic $X_\pi \in \mathfrak{g}$ such that*

$$\Theta_\pi(1+Y) = \deg(\pi) \cdot \widehat{\mu_{X_\pi}}(Y)$$

for all $Y \in V^{\text{reg}}_{(\rho(\pi)/2)+}$.

Remark 6.

1. This result is very similar to results Fiona Murnaghan has obtained in [10, 11, 12, 13]. In fact, the only new ingredient in the proof below is the control which the Moy-Prasad filtrations provide. This control allows us to specify the neighborhood where the equality is valid.

2. Jeff Adler and I have obtained a similar result in the tame case for all supercuspidal representations of $\mathrm{GL}_n(F)$ [1]. The differences are that we must use a truncated exponential map instead of the $X \mapsto (1+X)$ map, and the equality is only valid on $V_{\rho(\pi)+}$ in this case.

Proof. We need to construct (π, W) and then show that the equality is valid on the indicated range. We will show how this works in a very simple case.

Fix a positive integer m. Let $b = \varpi^{-2m} \cdot \Pi$. Let $E_b = F[b]$. Note that $\wp_{E_b}^k = E_b \cap \mathfrak{b}_k$ and $b \in \wp_{E_b}^{(1-2nm)} \setminus \wp_{E_b}^{(2-2nm)}$. Choose $\phi \in E_b^\times$ such that

$$\phi(1+T) = \Lambda\big(\mathrm{tr}(b \cdot T)\big)$$

for all $T \in \wp_{E_b}^{nm}$. Extend ϕ to $E_b^\times B_{nm}$ by

$$\phi(1+X) = \Lambda\big(\mathrm{tr}(b \cdot X)\big)$$

for all $X \in \mathfrak{b}_{nm}$. Put $\pi = \mathrm{Ind}_{E_b^\times B_{nm}}^G \phi$. Then π is a supercuspidal irreducible representation of G of depth $\rho(\pi) = (2mn-1)/n$.

Let X_π be a regular element in $b + \mathfrak{b}_{1010}$. (b is usually regular, but it will sometimes fail to be regular in positive characteristic.) Let $E = F[X_\pi]$. Note that $\wp_E^k = E \cap \mathfrak{b}_k$.

We now need to establish the equality between the character and the Fourier transform of the elliptic orbital integral on $V_{(\rho(\pi)/2)^+}^{\mathrm{reg}}$. Therefore, we need to show that

$$\Theta_\pi(1+X) = \deg(\pi) \cdot \widehat{\mu_{X_\pi}}(X)$$

for all $X \in \mathfrak{k}_m^{\mathrm{reg}}$. (I leave it as an exercise to show that $V_{((2nm-1)/2n)^+} = {}^G\mathfrak{k}_m$.)

Rader and Silberger [14] have extended a result of Harish-Chandra [4] to show that for $\gamma \in G^{\mathrm{reg}}$

$$\Theta_\pi(\gamma) = \deg(\pi) \cdot \int_{G/Z} dg^* \int_{B_0} dk\, \dot{\phi}({}^{gk}\gamma) \tag{7}$$

where dk is the normalized Haar measure of B_0. For $g \in G$, $\dot{\phi}(g)$ is $\phi(g)$ if $g \in E_b^\times B_{nm}$, and $\dot{\phi}(g)$ is zero otherwise.

On the other hand, from [5, Lemma 19], if $Y \in \mathfrak{g}^{\mathrm{reg}}$, then

$$\widehat{\mu_{X_\pi}}(Y) = \int_{G/Z} dg^* \int_{B_0} dk\, \Lambda\big(\mathrm{tr}(X_\pi \cdot {}^{gk}Y)\big).$$

This was originally proved in characteristic zero, but since we are dealing with $\mathrm{GL}_n(F)$, there is no difficulty in extending it to positive characteristic.

In [13, Lemma 4.1] an additional integration over B_0 is introduced to these two integral formulas. Using this reformulation, we must show that

$$\begin{aligned}
\Theta_\pi(1+X) &\equiv \deg(\pi) \cdot \int_{G/Z} dg^* \int_{B_0} dk_1 \int_{B_0} dk\, \dot{\phi}\big({}^{kgk_1}(1+X)\big) \\
&\stackrel{?}{=} \deg(\pi) \cdot \int_{G/Z} dg^* \int_{B_0} dk_1 \int_{B_0} dk\, \Lambda\big(\mathrm{tr}(X_\pi \cdot {}^{kgk_1}X)\big) \\
&\equiv \widehat{\mu_{X_\pi}}(X)
\end{aligned}$$

for all $X \in \mathfrak{k}_m^{\mathrm{reg}}$.

So it is enough to show, for fixed $(g, k_1) \in G \times B_0$, that if $Y = {}^{gk_1}X$, then

$$\int_{B_0} dk\, \dot{\phi}(1 + {}^kY) = \int_{B_0} dk\, \Lambda\big(\mathrm{tr}(X_\pi \cdot {}^kY)\big).$$

For support reasons, we can rewrite the left-hand side of this equality as

$$\begin{aligned}\int_{B_0} dk\, \dot{\phi}(1 + {}^kY) &= \begin{cases} \int_{B_0} dk\, \phi(1 + {}^kY) & \text{if } Y \in \mathfrak{b}_{nm} \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} \int_{B_0} dk\, \Lambda\big(\mathrm{tr}(b \cdot {}^kY)\big) & \text{if } Y \in \mathfrak{b}_{nm} \\ 0 & \text{otherwise} \end{cases}\end{aligned}$$

Because b and X_π are close enough, we may replace b with X_π in the last line. It is now enough to show that if $Y \notin \mathfrak{b}_{nm}$, then

$$\int_{B_0} dk\, \Lambda\big(\mathrm{tr}(X_\pi \cdot {}^kY)\big) = 0.$$

So, suppose that $Y \in \mathfrak{b}_t \smallsetminus \mathfrak{b}_{t+1}$ with $t < nm$. We know from §3.1 that

$$Y = {}^{gk_1}X \in {}^{gk}\mathfrak{k}_m \in \mathfrak{b}_{nm} + \mathcal{N}.$$

Since $t < m$, this implies that Y is *not* an element of $E + \mathfrak{b}_{t+1}$. Let $s = \lceil \frac{2nm-t}{2} \rceil$. Then

$$\int_{B_0} dk\, \Lambda\big(\mathrm{tr}(X_\pi \cdot {}^kY)\big) = \mathrm{const} \cdot \int_{B_0} dk \int_{\mathfrak{b}_s} dZ\, \Lambda\big(\mathrm{tr}(X_\pi \cdot {}^{(1+Z)k}Y)\big)$$

Fix $k \in B_0$. Then the inner integral becomes

$$\mathrm{const}' \cdot \int_{\mathfrak{b}_s} dZ\, \Lambda\big(\mathrm{tr}([X_\pi, {}^kY] \cdot Z)\big).$$

However, since ${}^kY$ is not an element of $E + \mathfrak{b}_{t+1}$, from [2] the bracket $[X_\pi, {}^kY]$ is not in $\mathfrak{b}_{(1-s)}$, and so the integral above is zero. □

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