Classical Sampling Theorem

Theorem $T, \Omega > 0$, $0 < T \leq \frac{1}{2\Omega}$, $f \in PW_\Omega$.

$$f = T \sum f(nT) \frac{\sin 2\pi\Omega(t-nT)}{\pi(t-nT)}.$$

Proof. $f \leftrightarrow F$

$$f(t) \underset{INV}{\overset{FT}{=}} \int F(\gamma) e^{2\pi i t\gamma} d\gamma =$$

$$\int_{-\Omega}^{\Omega} G(\gamma) e^{2\pi i t\gamma} d\gamma \underset{\int FS}{\overset{FS}{=}} \sum c_n \int_{-\Omega}^{\Omega} e^{2\pi i(t-nT)\gamma} d\gamma.$$

$$c_n = \hat{G}[n], \quad G \in L^1(\mathbb{T}_{1/(2T)}),$$

$$G(\gamma) = \begin{cases} F(\gamma), & |\gamma| \leq \Omega \\ 0, & \Omega < |\gamma| \leq \frac{1}{2T}. \end{cases}$$

$$S(G) = \sum c_n e^{-\pi i n\gamma(2T)} \Rightarrow$$

$$c_n = T f(nT). \quad \blacksquare$$