

## PSF

Given  $f : \mathbb{R} \rightarrow \mathbb{C}$ . Define the map

$$f \mapsto f_T^\circ = T \sum \tau_{-mT} f,$$

so that  $f_T^\circ : \mathbb{R}/T\mathbb{Z} \rightarrow \mathbb{C}$ . Assume

$$(1) \quad f_T^\circ(x) = \sum \overset{2\pi i x m / T}{\overset{\circ}{F}_T[m]} e^{2\pi i x m / T},$$

where  $\overset{\circ}{F}_T$  is the sequence of Fourier coefficients of  $f_T^\circ$ . Compute

$$(2) \quad \overset{\circ}{F}_T[m] = \hat{f}\left(\frac{m}{T}\right).$$

Combine (1) and (2) to obtain

$$(PSF) \quad T \sum f(t+mT) = \sum \hat{f}\left(\frac{m}{T}\right) e^{2\pi i t m / T},$$

$$(PSF) \quad T \sum f(mT) = \sum \hat{f}\left(\frac{m}{T}\right),$$

$$(PSF) \quad T \sum \delta_{mT} \longleftrightarrow \sum \delta_{m/T},$$

great formulas which are sometimes true!