Stat 400 **PRACTICE FINAL EXAM** **Dec. 2, 2005**

J.Millson

1. Let X be a discrete random variable with probability mass function p given by

$$p(1) = 1/3,\ p(2) = 1/3,\ p(3) = 1/3.$$

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(a) Find $E(X)$.
(b) Find $E(X^2)$.
(c) Find $V(X)$.
(d) Find $F(x)$, the cumulative distribution function of X.
(e) Make the change of variable $Y = X - 1$. Find the probability mass function of the new random variable Y.

(20 points)

2. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} 2(1-x),\ 0 \leq x \leq 1, \\ 0, otherwise. \end{cases}$$

(a) Find E(X).
(b) Find V(X).
(c) Find $F(x)$, the cumulative distribution function of X.
(d) Find the median of X.
(e) Find the 75-th percentile of X

(20 points)

3. Two couples (Jack and Jill and Dick and Jane) go to the movies and are seated randomly in four adjacent seats. What is the probability some husband sits beside his wife?

(10 points)

4 (a) Suppose that X and Y are independent random variable defined on the same sample space, that X has gamma distribution distribution with parameters α_1 and β and Y has gamma distribution with parameters α_2 and β. How is the sum $X + Y$ distributed? (Hint: Use moment-generating functions)

(b) Suppose W has gamma distribution with parameters $\alpha = 100$ and $\beta = 1$. Use the version of the Central Limit Theorem that says the sum of n independent identically distributed random variables can be normally approximated provided $n > 30$ to prove that W can be normally approximated. (Hint: Use (a) to show that W is the sample total for a random sample of size 100 from a different gamma distribution.)

(c) Carry out the normal approximation from (b) to compute an approximate value for $P(W \leq 110)$.

(20 points)

5. A sample of 26 offshore oil workers took part in a simulated escape exercise. The resulting 26 escape times were recorded with a sample mean of 24.36 and a sample standard deviation of 370.69.

(i) Calculate a 95% lower-tailed confidence interval (upper confidence bound) for population mean escape time.

(ii) Calculate a 95% lower-tailed prediction interval (upper prediction bound) for a single additional worker.

(20 points)

6. Let $X_1, X_2, \cdots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2. We wish to predict the next observation X_{n+1}. Let $\overline{X} = \frac{X_1+\cdots+X_n}{n}$ be the sample mean for the first n observations and $S^2 = \frac{\Sigma_{i=1}^n (X_1-\overline{X})^2}{n-1}$ be the sample variance for the first n-observations. Assume the theorem that $T = \frac{\overline{X}-X_{n+1}}{\sqrt{\frac{n+1}{n}}S}$ has t distribution with $n-1$ degrees of freedom and prove that the random interval

$$(-\infty,\ \overline{X} + t_{\alpha,n-1}\sqrt{\frac{n+1}{n}}S)$$

is a $100(1-\alpha)\%$ prediction interval for the next observation X_{n+1}.

(10 points)