Mathematics 241, Section 2101
Exam 3
November 14, 2002

Instructions: Please show all work, as answers without supporting work will receive no credit. Answer each of the 4 problems on a seperate answer sheet.

1. (25 points)
Set up and evaluate a double integral computing the **surface area** of a sphere of radius 11. If you wish you may solve this problem by parameterizing the sphere using a special case of spherical coordinates; this may avoid a messy computation.

2. (30 points) The following 3 regions all have mass density

$$\delta(x, y, z) = 10 + 4x + 3y + 2z.$$

For each region, set up but do **not** evaluate, an iterated triple integral computing the total mass of the respective region. You must use spherical coordinates for one integral, cylindrical coordinates for another, and rectangular coordinates for the remaining one. **Make your choices wisely**. If you do not know how to find the total mass, you may instead set up integrals giving the volume of each region, but you will then lose 10 points.

(a) D_1 is the region bounded above by the sphere $x^2 + y^2 + z^2 = 16$ and bounded below by the plane $z = \sqrt{7}$.

(b) D_2 is the region in the octant with $x > 0$, $y > 0$, and $z < 0$, bounded below by the sphere $x^2 + y^2 + z^2 = 10$, bounded above by the lower nappe of the cone $z^2 = x^2 + y^2$, and bounded on the sides by the planes $y = \sqrt{3}x$ and $x = \sqrt{3}y$ (i.e., $y \leq \sqrt{3}x$ and $x \leq \sqrt{3}y$).

(c) D_3 is the region inside the sphere $x^2 + y^2 + z^2 = 64$ and between the upper and lower nappes of the cone $3z^2 = x^2 + y^2$ (Note: by "between" I mean "below the upper nappe and above the lower nappe.")

3. (25 points) Evaluate

$$\iint_R x^6 y e^{x^4 y}\, dA,$$

where R is the region in the first quadrant bounded by the curves $x^4 y = 2$, $x^4 y = 4$, $xy^{1/3} = 1$, and $xy^{1/3} = 3$.

4.
(a) (15 points) Reverse the order of integration for the iterated integral

$$\int_{-1}^{1} \int_{x-1}^{1-x} 2x + y + 4\, dy\, dx.$$

You do **not** need to evaluate the result, you only need to find the correct limits of integration.
(b) (5 points) If we interprete the integrand above as a mass density for the region described by the limits, then where in the region is the density the largest.