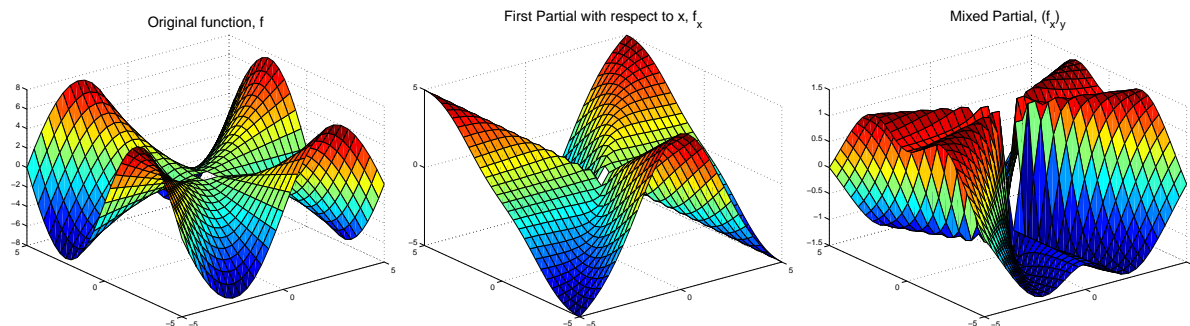


MATH 241, Section 2101
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Here are the relevant plots that exhibit a function whose mixed partials f_{xy} and f_{yx} are not continuous.



The left figure is the graph of $f(x, y) = \frac{x^3y - xy^3}{x^2 + y^2}$. Notice as you approach the origin along the y -axis, the slopes in the x -direction are decreasing like $-y$. That is, at the point $(0, 2, 0)$ the slope in the x -direction is -2 , whereas at the point $(0, 1, 0)$ the slope in the x -direction is -1 . The center figure records these slopes as it is the graph of the function f_x . The right figure exhibits the change in these slopes with respect to the y -direction since it is the graph of the function $(f_x)_y$. Therefore, along the y -axis, $(f_x)_y(0, y) = -1$. This -1 translates into a clockwise rotation of the slopes in the x -direction as we look down the positive y -axis and let $y \rightarrow 0^+$ in the left figure.

A different thing is happening as you approach the origin along the x -axis. Along this approach, the slopes in the x -direction are changing like y . So in particular, $f_x(x, 0) = 0$. We can see this by looking at the graph of f_x . Therefore, along the x -axis, $(f_x)_y(x, 0) = 1$. This $+1$ translates to a clockwise rotation of the slopes in the y -direction as we look down the positive x -axis and let $x \rightarrow 0^+$.

If the mixed partials are to equal, then as we look down the positive x -axis and let $x \rightarrow 0^+$, we should see the slopes rotating in one direction and when we look down the positive y -axis and let $y \rightarrow 0^+$, we should see the slopes rotating in the opposite direction. Note this is the case with the saddle.

