Mathematics 241
First Exam
Dr. Rosenberg
Friday, February 28, 2003

Instructions. Answer each question **on a separate answer sheet**. *Be sure your name, section number, and problem number are on each answer sheet.* The point value of each problem is indicated. The exam is worth a total of 100 points. In problems with multiple parts, whether the parts are related or not, the parts are graded independently of one another. Be sure to go on to subsequent parts even if there is some part you cannot do. Please leave answers such as $5\sqrt{2}$ in terms of radicals and **do not convert to decimals**. You are allowed use of a non-programmable calculator and one sheet of notes.

1. (30 points) In this problem, you may make use of the following tables of dot and cross products:

$$(-1,-2,6)\cdot(0,4,1) = -2, \quad (-1,-2,6)\cdot(1,0,1) = 5, \quad (0,4,1)\cdot(1,0,1) = 1.$$

$$\begin{aligned}(-1,-2,6)\times(0,4,1) &= (-26,1,-4),\\ (-1,-2,6)\times(1,0,1) &= (-2,7,2),\\ (0,4,1)\times(1,0,1) &= (4,1,-4).\end{aligned}$$

a) Show that the points $P_1 = (-1,-2,6)$, $P_2 = (0,4,1)$, $P_3 = (1,0,1)$ determine a unique plane $\mathcal{P}$, and determine the equation of $\mathcal{P}$.
b) Find the area of the triangle with vertices P_1, P_2, and P_3.

2. (10 points) Let $\mathcal{P}_1$ and $\mathcal{P}_2$ be the planes with equations

$$3x - 4y + z = 2, \qquad 3x + 2y - z = 7.$$

Find symmetric equations of the line $\mathcal{L}_1$ where $\mathcal{P}_1$ and $\mathcal{P}_2$ intersect. You may use the fact that $(3,-4,1)\times(3,2,-1) = (2,6,18)$.

3. (20 points) a) Find a vector form of the equation of the line $\mathcal{L}_2$ through $P_4 = (7,0,1)$ perpendicular to the plane $\mathcal{P}_3$ with equation $x + y + z = 11$.
b) Determine the *closest point* to P_4 on $\mathcal{P}_3$.

4. (40 points) Answer the following questions about the curve with parameterization $\mathbf{r}(t) = (3\cos t, 3\sin t + \cos(t/2))$. **Refer to the MATLAB session on the other sheet.**
a) The curve is *periodic*; that is, there is a positive value of T such that $\mathbf{r}(t) = \mathbf{r}(t+T)$ for all t. What is the smallest period, i.e., the smallest value of T for which this holds?
b) The curve has two *nodes*, i.e., self-intersection points. Find their exact coordinates (using the MATLAB session).
c) What are the approximate values of the minimum and maximum radius of curvature, and roughly where on the curve are the points where the minimum and maximum curvature occur?
d) Does the curve have any inflection points? If so, where are they located? If not, explain why not.

```
>> syms t
>> veclength=inline('sqrt(v*transpose(v))')

veclength =

     Inline function:
     veclength(v) = sqrt(v*transpose(v))

>> curve = [3*cos(t), 3*sin(t) + cos(t/2)];
>> ezplot(curve,[0,4*pi])
```
(output shown as Figure 1)
```
>> solve(curve(2))

ans =

[                         pi]
[                        -pi]
[     -2*atan(1/35*35^(1/2))]
[ 2*atan(1/35*35^(1/2))-2*pi]

>> v=diff(curve); a=diff(v); vca=cross([v,0],[a,0]);
>> vca=simplify(vca(3))

vca =

9-3/2*sin(1/2*t)*cos(1/2*t)^2+3/2*sin(1/2*t)

>> k=simplify(vca/veclength(v)^3);
>> ezplot(k, [0, 4*pi])
```
(output shown as Figure 2)

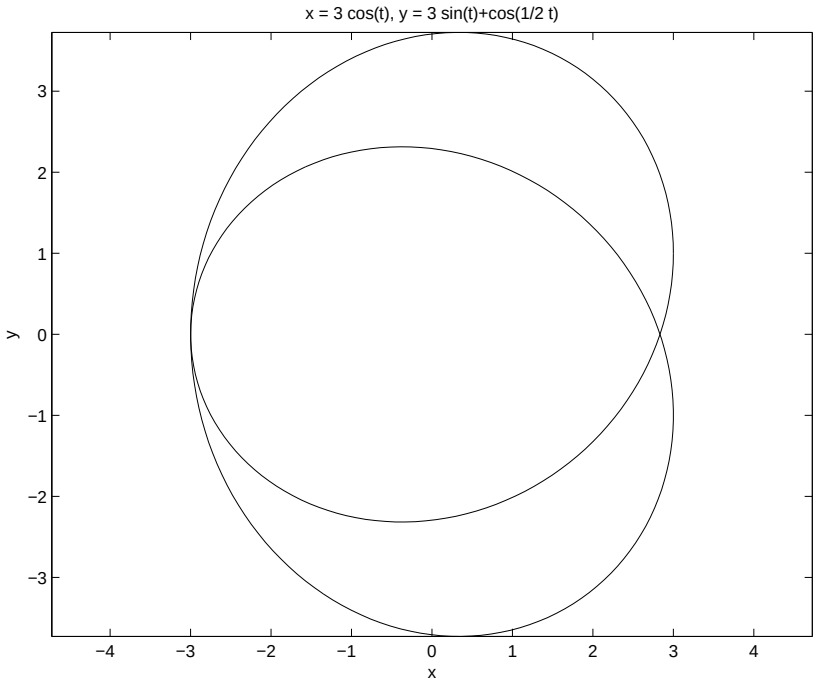

Figure 1

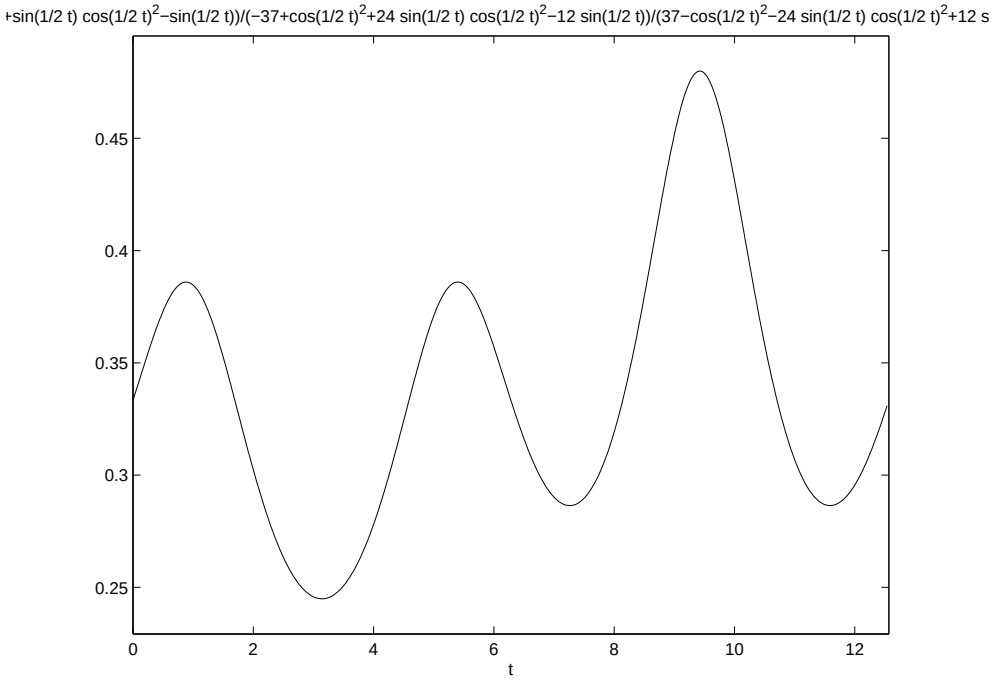

Figure 2