Mathematics 241
Third Exam
Dr. Rosenberg
Friday, May 2, 2003

Instructions. Answer each question **on a separate answer sheet**. *Be sure your name, section number, and problem number are on each answer sheet.* The point value of each problem is indicated. The exam is worth a total of 100 points. In problems with multiple parts, whether the parts are related or not, the parts are graded independently of one another. Be sure to go on to subsequent parts even if there is some part you cannot do. Please leave answers such as $5\sqrt{2}$ in terms of radicals and **do not convert to decimals**.
You are allowed use of a non-programmable calculator and one sheet of notes.

1. (25 points) Evaluate by reversing the order of integration:

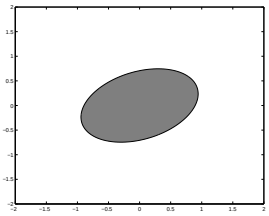
$$\int_0^1 \int_0^{\tan^{-1}(x)} \cos y \, dy \, dx.$$

You will receive only 15 out of the 25 points if you correctly evaluate **without** reversing the order of integration.

2. (20 points) **Set up** (with explicit limits of integration), but **do not evaluate**, a multiple integral for the volume of the solid inside the hyperboloid $x^2 + y^2 - z^2 = 1$ and between the planes $z = -1$ and $z = 1$.

3. (25 points) **Set up** (with explicit limits of integration), but **do not evaluate**, a multiple integral for the area of the region shown in the picture, bounded by the ellipse

$$(x + 2y)^2 + 4(x - y)^2 = 4.$$



4. (30 points, 10 for (a) and 20 for (b))
(a) Show from the Divergence Theorem that if D is a solid region in $\mathbb{R}^3$ with smooth boundary Σ, and if $\mathbf{n}$ is the unit outward normal to Σ, then the volume of D is given by the flux integral

$$\iint_\Sigma z \, \mathbf{n} \cdot \mathbf{k} \, dS.$$

Important warning: This is basically a one-line calculation. If you're trying anything complicated, you are on the wrong track.
(b) Write an explicit iterated integral (with explicit limits of integration) for the flux integral of (a), in the special case where Σ is the unit sphere $x^2 + y^2 + z^2 = 1$. You need not evaluate your integral (though if you have extra time, you might want to check that it indeed evaluates to the volume of D).