MATH 602 (Homological Algebra) Assignment #8: Cohomology and Extension Theory of Groups

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due Friday, April 27, 2007

1. Let p be a prime and let $\mathbb{F}_p$ denote the field of p elements. Let C_p denote a cyclic group of order p, with trivial action on $\mathbb{F}_p$. By last week's homework, $H^2(C_p; \mathbb{F}_p) \cong \mathbb{F}_p$. Explain how this fact and the classification of extensions of C_p by $\mathbb{F}_p$ matches up with the classification theorem for groups of order p^2. (Recall all such groups are abelian!)

2. Show that $H^2(C_p \times C_p; \mathbb{F}_p)$ has dimension 3 over $\mathbb{F}_p$. By the Künneth Theorem — you are allowed to use this; see Exercise 6.1.10(2) in Weibel, page 166 — this cohomology group can be identified with

$$H^2(C_p; \mathbb{F}_p) \otimes H^0(C_p; \mathbb{F}_p) \oplus H^1(C_p; \mathbb{F}_p) \otimes H^1(C_p; \mathbb{F}_p) \oplus H^0(C_p; \mathbb{F}_p) \otimes H^2(C_p; \mathbb{F}_p) .$$

3. Use the result of (2) to classify central extensions of $C_p \times C_p$ by C_p. Show there is a nonabelian such extension which can be realized as the *Heisenberg group* over $\mathbb{F}_p$, the group of 3×3 matrices over $\mathbb{F}_p$ of the form

$$\begin{pmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix} .$$

4. When $p = 2$, there are two isomorphism classes of nonabelian groups of order 8, represented by the dihedral group and the quaternion group. Where do they fit into this classification?