MATH 748R, Spring 2012
Homotopy Theory
Homework Assignment #5: Eilenberg-MacLane Spaces and Obstruction Theory

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due Friday, March 30, 2012

1. Use the theory of Postnikov systems to classify (up to homotopy equivalence) all CW complexes X with $\pi_2(X) \cong \pi_3(X) \cong \mathbb{Z}$ and all other homotopy groups 0. (It should turn out there is a one-parameter family of such X's; how do you distinguish them?)

2. Classify up to homotopy all maps $K(\mathbb{Z}, 2) \to K(\mathbb{Z}, 2)$, and show that if you think of $K(\mathbb{Z}, 2)$ as BS^1, that they all arise as $B\phi$ for some homomorphism of topological groups $\phi\colon S^1 \to S^1$.

3. Classify up to homotopy all maps $K(\mathbb{Z}/2, 1) \to K(\mathbb{Z}, 2)$, and show that if you think of $K(\mathbb{Z}/2, 1)$ as $B(\mathbb{Z}/2)$, that they all arise as $B\phi$ for some group homomorphism $\phi\colon (\mathbb{Z}/2) \to S^1$.

4. Let X be a connected CW complex with a distinguished 0-cell as basepoint. Show that if G is a discrete group, any homomorphism $\pi_1(X) \to G$ can be realized by a unique homotopy class of based maps $X \to K(G, 1)$. (This is a slight variant of a theorem proved in class.)