# Fermat's Challenge Problem on $61x^2 + 1 = y^2$

```
numlib::contfrac(sqrt(61))
```

$$7+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{14+\ldots}}}}}}}}}}}$$

```
a0 := numlib::sqrt2cfrac(61)[1];
a1 := numlib::sqrt2cfrac(61)[2]
```

[7]

[1, 4, 3, 1, 2, 2, 1, 3, 4, 1, 14]

From this we can get the series of "convergents", from which we can look for a solution to $61x^2 + 1 = y^2$. The recursive relations are $h_n = a_n h_{n-1} + h_{n-2}$, $k_n = a_n k_{n-1} + k_{n-2}$, starting with
$a_0 = 7$, $h_{-2} = 0$, $h_{-1} = 1$, $k_{-2} = 1$, $k_{-1} = 0$.

```
hprev := 1: hcurr := 7: kprev := 0: kcurr := 1:
for i from 1 to 22 do
   j := i mod 11:
   if j =0 then j := 11 end_if:
   h1 := hcurr: k1 := kcurr:
   hcurr := a1[j]*h1+hprev:
   kcurr := a1[j]*k1+kprev:
   print([hcurr, kcurr, hcurr^2 - 61*kcurr^2]);
   hprev := h1: kprev := k1:
end_for
```

[8, 1, 3]

[39, 5, −4]

[125, 16, 9]

[164, 21, −5]

[453, 58, 5]

[1070, 137, −9]

[1523, 195, 4]

[5639, 722, −3]

[24079, 3083, 12]

[29718, 3805, −1]

[440131, 56353, 12]

[469849, 60158, −3]

[2319527, 296985, 4]

[7428430, 951113, −9]

[9747957, 1248098, 5]

[26924344, 3447309, −5]

```
[63596645, 8142716, 9]
[90520989, 11590025, -4]
[335159612, 42912791, 3]
[1431159437, 183241189, -12]
[1766319049, 226153980, 1]
[26159626123, 3349396909, -12]
226153980
```

So we found the smallest solution to Fermat's challenge:

```
1766319049^2 - 61*226153980^2
1
```

Not something you'd get by trial and error!