The “New Algebra” of François Viète

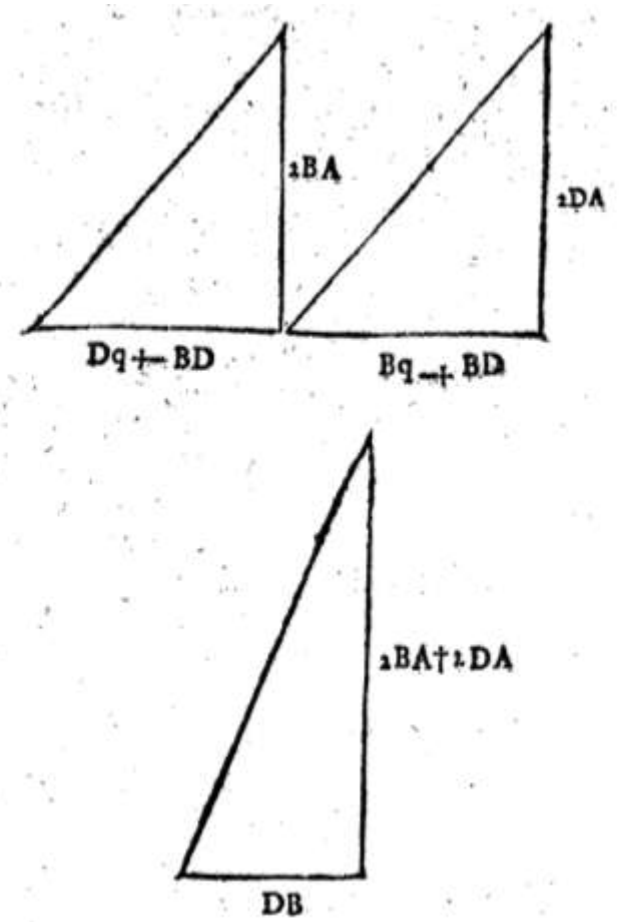
Book IV [translated by J. Rosenberg]

Zetetic [Inquiry, from Greek zētētikos, from zēteō to seek] XI

To find three [distinct] right triangles [with integral sides] with the same area.

Let the altitude of one of the triangles be 2BA, the base $D^2 + BD$; the altitude of the second 2DA, the base $B^2 + BD$; the altitude of the third 2BA + 2DA, the base DB. Then the three triangles all have the same area, namely

$$BA(D^2 + BD) = DA(B^2 + BD)=(BA + DA)(DB)=ABD(B + D).$$



By the previous zetetic, the quantities B and D can be chosen so that $B^2 + D^2 + BD$ is a perfect square, say A^2. [To do this, write $B^2 + D^2 + BD = (D + C)^2 = D^2 + 2DC + C^2$, so $B^2 – C^2 = (B – C)(B + C) = D(2C– B)$. Take $B + C = D$ and $B – C = 2C– B$, or $2B = 3C$, $D = 5B/3$.] Then the base of the first triangle is $D^2 + BD = A^2 – B^2$, the base of the second is $B^2 + BD =$

$A^2 – D^2$, and of the third is $BD = (B + D)^2 – A^2$. Now we have

$$(A^2 – B^2)^2 + (2BA)^2 = A^4 – 2A^2B^2 + B^4 + 4A^2B^2 = A^4 + 2A^2B^2 + B^4 = (A^2 + B^2)^2$$

so the hypotenuse of the first triangle is $A^2 + B^2$. Similarly the hypotenuse of the second triangle is

$A^2 + D^2$, and the hypotenuse of the third is $(B + D)^2 + A^2$. Thus we've accomplished what was required. For example, take $B = 3$, $D = 5$. Then $B^2 + D^2 + BD$ is $9 + 25 + 15 = 49 = 7^2$, so take $A = 7$.

The first triangle has base $7^2 – 3^2 = 40$, altitude $2·7·3 = 42$, hypotenuse $7^2 + 3^2 = 58$.

The second triangle has base $7^2 – 5^2 = 24$, altitude $2·7·5 = 70$, hypotenuse $7^2 + 5^2 = 74$.

The third triangle has base $8^2 – 7^2 = 15$, altitude $2·7·8 = 112$, hypotenuse $7^2 + 8^2 = 113$.