Homework #1

Math 446 Due Tuesday, February 9

1. Prove parts (a), (d), (f) of Proposition 1.15 of the text.

2. Suppose that $(X, \leq)$ is a linear order. Prove that $(FIN(X), \leq_{lex})$ is a linear order.

3. Suppose that $(X, \leq)$ is a linear order. For every $n \in \mathbb{N}$, let $L_n(X)$ be the subset of $FIN(X)$ consisting of all sequences from X of length at most n.

   a. Let $\leq$ denote the usual ordering on $\mathbb{N}$. True or false: $L_1(\mathbb{N})$ is a cofinal subset of $(FIN(\mathbb{N}), \leq_{lex})$. Prove your answer is correct.

   b. Prove, by induction on n, that if $(X, \leq)$ is a well order, then so is $(L_n(X), \leq_{lex})$.

   c. Let $\leq$ denote the usual ordering on $\mathbb{N}$. True or false: $(FIN(\mathbb{N}), \leq_{lex})$ is a well-order. Prove your answer is correct.

   d. Let $\leq$ denote the usual ordering on $\mathbb{N}$. Find a subordering of $(\mathbb{Q}, \leq)$ that is isomorphic to $L_3(\mathbb{N})$.

4. Find a subordering of $(\mathbb{Q}, \leq)$ that is a well ordering with exactly 5 limit points.