Homework #1

Math 446 Due Tuesday, February 7

1. Prove parts (a), (d), (f) of Proposition 1.15 of the text.

2. Suppose that $(X, \leq)$ is a linear order. Prove that $(FIN(X), \leq_{lex})$ is a linear order.

Definition: Two linear orders $(X, \leq_X)$ and $(Y, \leq_Y)$ are *isomorphic* if there is a 1-1, onto $f : X \to Y$ that is *order-preserving* i.e., for $a, b, \in X$,

$$a \leq_X b \qquad \text{if and only if} \qquad f(a) \leq_Y f(b)$$

3. Suppose that $(X, \leq)$ is a linear order. For every $n \in \mathbb{N}$, let $L_n(X)$ be the subset of $FIN(X)$ consisting of all sequences from X of length at most n.
   a) Let $\leq$ denote the usual ordering on $\mathbb{N}$. Find a subordering of $(\mathbb{Q}, \leq)$ that is isomorphic to $L_1(\mathbb{N})$.
   b) Let $\leq$ denote the usual ordering on $\mathbb{N}$. Find a subordering of $(\mathbb{Q}, \leq)$ that is isomorphic to $L_3(\mathbb{N})$.
   c) Prove, by induction on n, that if $(X, \leq)$ is any well order, then so is $(L_n(X), \leq_{lex})$.
   d) Let $\leq$ denote the usual ordering on $\mathbb{N}$. True or false: $(FIN(\mathbb{N}), \leq_{lex})$ is a well-order. Prove your answer is correct.

4. Find a subordering of $(\mathbb{Q}, \leq)$ that is a well ordering with exactly 5 limit points.