Math 406, Introduction to Number Theory

# MIDTERM EXAM

**1)** Find all integer solutions to the equation $40x + 64y = 56$.

**2)** Find all natural numbers $n > 1$ such that (i) $n$ is the square of an integer and (ii) $1453 \equiv 2713 \pmod{n}$.

**3)** Consider the number $n = 23x45678y$, where $x$ and $y$ are unknown digits. We know that $n$ is a multiple of 15. Find all such numbers $n$.

**4)** Use the Euclidean algorithm to compute $\gcd(77777777, 77777)$.

**5)** Suppose that $a$ and $b$ are integers such that $(a, b) = 1$. If $c$ divides $a + b$, prove that $(a, c) = 1$.

**6)** The three most recent appearances of Halley's comet were in the years 1835, 1910, and 1986, while the next appearance will be in 2061. Prove that
$$1835^{1910} + 1986^{2061} \equiv 0 \pmod{7}.$$

**8)** For any two integers $a$ and $b$, prove that $35 \mid ab(a^{12} - b^{12})$. [Hint: $35 = 5 \cdot 7$]