MATH 600: ABSTRACT ALGEBRA (L. WASHINGTON)
EXAM #1, OCTOBER 20, 1994

1. Let G be a finite group and let $g \in G$. Let $S = \{xgx^{-1} | \, x \in G\}$. Show that $|S|$ divides $|G|$.

2. Let H and K be subgroups of a group G. Let $HK = \{hk | \, h \in H, \, k \in K\}$. Show that HK is a subgroup of G if and only if $HK = KH$.

3. Let G be a finite non-abelian simple group. Let p be a prime dividing $|G|$ and let m be the number of Sylow p-subgroups of G. Show that $|G|$ divides $m!$.

4. Let G be a group of order $5 \cdot 11 \cdot 17$. Suppose G contains an element of order 55. Show that G is cyclic.

5. (a) Find all normal subgroups of $A_5 \times \mathbb{Z}/2\mathbb{Z}$.
(b) Show that $S_5 \not\cong A_5 \times \mathbb{Z}/2\mathbb{Z}$.
(c) Show that S_5 can be expressed in the form $A_5 \rtimes_\phi \mathbb{Z}/2\mathbb{Z}$.