MATH 600: ABSTRACT ALGEBRA (L. WASHINGTON)
SAMPLE EXAM #2

1. Explicitly construct a field with 27 elements.

2. Let R be a unique factorization domain and let S be a non-empty set of principal ideals of R. Show that there is an ideal $I \in S$ that is maximal for S (that is, there is no $J \in S$ with $I \subsetneq J$).

3. Let R and S be commutative rings with 1 and let $f : R \to S$ be a homomorphism satisfying $f(1) = 1$.
(a) Let P be a prime ideal of S. Show that $f^{-1}(P)$ is a prime ideal of R.
(b) Assume f is surjective and let M be a maximal ideal of S. Show that $f^{-1}(M)$ is a maximal ideal of R.

4. Let R be a commutative Noetherian ring and let I be an ideal of R. Let $J = \{r \in R | r^n \in I \text{ for some integer } n \geq 1\}$ (n may depend on r).
(a) Show that J is an ideal of R.
(b) Show that there is an integer $N \geq 1$ such that $r^N \in I$ for all $r \in J$ (N is independent of r).
(c) Give an example of a non-Noetherian ring R and an ideal I such that N does not exist.

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