

Homework #5 (Due Friday, March 12)

p. 604: 8.46, 8.47, 8.49, 8.50

I. Let R be a commutative ring with identity and let I and J be ideals of R . Then R/I and R/J are R -modules in the natural way.

(a) Show that every element of $R/I \otimes_R R/J$ can be written in the form $(r + I) \otimes (s + J)$.

(b) Show that the map

$$R/I \otimes_R R/J \rightarrow R/(I + J)$$

$$\sum (r_i + I) \otimes (s_i + J) \mapsto \sum r_i s_i + (I + J)$$

is a well-defined isomorphism.

(c) Show that $\mathbf{Z}_m \otimes_{\mathbf{Z}} \mathbf{Z}_n \simeq \mathbf{Z}_d$, where $d = \gcd(m, n)$.