Homework 7. Due Thursday Dec. 9.

1. **(10 pts)** Consider a random graph with a degree distribution where each vertex has degree 1 with probability $p_1 = p$ or degree 3 with probability $p_3 = 1 - p$. Assume that the number of vertices is very large.

   (a) Write out the criterion for existence of the giant component in terms of p_1 and p_3.

   (b) Find an analytic expression for the generating function $H_0(x)$ for component size distribution: $H_0(x) = \sum_{s=1}^{\infty} P_s x^s$, where P_s is the probability for a randomly picked vertex to lie in a connected component of size s. *Hint: you will need to select a root of a quadratic equation. When you are making this selection, think of what should be $H_1(x)$ as $x \to 0$.*

   (c) Choose values for $p_1 > 0$ and $p_3 > 0$ so that there is no giant component. Use Cauchy's integral formula to extract the component size distribution from $H_0(x)$:

   $$P_s = \frac{1}{2\pi i} \oint \frac{H_0(z)dz}{z^{s+1}}.$$

   Hint: use the unit circle around the origin and the composite trapezoidal rule to evaluate the integrals.

   (d) Repeat the last task for a combination of values p_1 and p_3 so that there is a giant component. Also find the fraction of nodes in the giant component.

2. **(10 pts)**

   (a) Take values for p_1 and p_3 from the previous problem for which there is no giant component. Take the number of vertices $n = 10^4$ and generate a random graph as described in Newman, Strogatz and Watts (2001) (section G). *After your randomly match the stubs, remove self-loops and repeated edges.* Use the depth-first search algorithm to find the sizes of connected components. Estimate probabilities for component-size distribution from these data and plot them in the same figure as probabilities for component sizes obtained using Cauchy's formula.

   Repeat the task for p_1 and p_3 from the previous problem for which there is a giant component. Also compare the fraction of nodes in the largest component with the fraction in the giant component estimated in the previous problem.