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The main theorem of the paper (Theorem 1) is true as stated. However, the proof requires a modification. We thank Lili He for bringing this to our attention.

Theorems 2 and 2.2 are not true as stated for null forms involving time derivatives, such as Q_{0i} and Q_0. They have to be modified by including the terms $\|F\|_{L^2(dt)L^3(dx)}$ and $\|G\|_{L^2(dt)L^3(dx)}$ in the right hand side of the equation. Thus, if $\Box\phi = F$ with Cauchy data f_0, f_1 and $\Box\psi = g$ with Cauchy data g_0, g_1 Theorems 2 and 2.2 should read

$$\int_0^T \int_{\mathbb{R}^3} |DQ(\phi,\psi)|^2 dxdt$$
$$\leq c \left(\|f_0\|_{H^2(\mathbb{R}^3)} + \|f_1\|_{H^1(\mathbb{R}^3)} + \int_0^T \|\nabla F(t,\cdot)\|_{L^2(\mathbb{R}^3)} dt + \|F\|_{L^2([0,T])L^3(dx)} \right)^2$$
$$\times \left(\|g_0\|_{H^2(\mathbb{R}^3)} + \|g_1\|_{H^1(\mathbb{R}^3)} + \int_0^T \|\nabla G(t,\cdot)\|_{L^2(\mathbb{R}^3)} dt + \|G\|_{L^2([0,T])L^3(dx)} \right)^2$$

The theorems are still true, as stated, for the null forms Q_{ij}.

The problem is with the time derivative for the formula in the middle of page 1237. This has to be modified (for Q_0) to

$$\partial_t Q_0(\phi,\psi)(t,\cdot)$$
$$= F(t,\cdot)\partial_t\psi(t,\cdot) + G(t,\cdot)\partial_t\phi(t,\cdot) + \int_0^t \int_0^t \partial_t Q_0 \left(R(t-\tau)F(\tau,\cdot), R(t-\sigma)G(\sigma,\cdot) \right) d\tau d\sigma$$

This was discovered by Lili He.

The originally stated estimate (without the terms $\|F\|_{L^2(dt)L^3(dx)}$, $\|G\|_{L^2(dt)L^3(dx)}$ is true for the last term. However, the estimate is not true for the first two terms. For instance, if $f_0 = 0$, $f_1 = 0$,

$$\|F\partial_t\psi\|_{L^2([0,T]\times\mathbb{R}^3)}$$
$$\leq c\|F\|_{L^1[0,T]H^1(\mathbb{R}^3)} \times \left(\|g_0\|_{H^2(\mathbb{R}^3)} + \|g_1\|_{H^1(\mathbb{R}^3)} + \int_0^T \|G(t,\cdot)\|_{H^1(\mathbb{R}^3)} dt \right)$$

cannot be true.

For this reason, we dominate

$$\|F\partial_t\psi\|_{L^2([0,T]\times\mathbb{R}^3)} \le \|F\|_{L^2([0,T])L^3(\mathbb{R}^3)}\|\nabla\partial_t\psi\|_{L^\infty([0,T]L^2(\mathbb{R}^3)}$$
$$\le \|F\|_{L^2([0,T])L^3(\mathbb{R}^3)}\left(\|g_0\|_{H^2(\mathbb{R}^3)} + \|g_1\|_{H^1(\mathbb{R}^3)} + \int_0^T \|\nabla G(t,\cdot)\|_{L^2(\mathbb{R}^3)}dt\right)$$

The statement of our main theorem (Theorem 1) is not affected by this modification. The proof of Theorem 1 requires only a minor change.

Recall our original definitions of X_1, X_2, E_2,

$$X_1^2(t) = \int_0^t \|DQ(\phi,\phi-\psi)\|_{L^2}d\tau$$
$$X_2^2(t) = \int_0^t \|DQ(\psi,\phi-\psi)\|_{L^2}d\tau$$
$$E_2(\phi)(t) = \sum_{0\le|A|\le 2}\|D^A\phi(t,\cdot)\|_{L^2(\mathbb{R}^3)}$$

To estimate the newly introduced terms, we use Holder's inequality and the Sobolev estimate to get

$$\|Q(\phi,\phi-\psi)(t,\cdot)\|_{L^3(\mathbb{R}^3)} \le CE_2(\phi)(t)E_2(\phi-\psi)(t)$$

thus, for solutions of the equation (3.1),

$$\|\Box(\phi-\psi)(t,\cdot)\|_{L^3(\mathbb{R}^3)} \le CE_2(\phi)(t)E_2(\phi-\psi)(t)$$

and

$$\|\Box(\phi-\psi)\|_{L^2[0,t]L^3(\mathbb{R}^3)} \le C\left(\int_0^t E_2^2(\phi-\psi)(\tau)d\tau\right)^{\frac{1}{2}}$$

Thus, when we apply the modified version of Theorem 2.2 to ϕ and $\phi-\psi$, we have the modification of (3.12)

$$X_1(t) \le C\int_0^t \|\nabla\Box(\phi-\psi)(\tau,\cdot)\|_{L^2}d\tau + \left(\int_0^t E_2^2(\phi-\psi)(\tau)d\tau\right)^{\frac{1}{2}}$$

which does not affect (3.8), the main estimate on which the uniqueness proof is based.

It is also possible to modify the statement and proof of Theorem 1 by using space-time norms involving only space derivatives.

Similar modifications apply to the existence part.

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