G2D2-2019 LECTURE SERIES
SYMBOLIC DYNAMICS AND THE STABLE ALGEBRA OF MATRICES

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We give an exposition of the relationships between algebraic invariants of matrices and dynamical invariants of the symbolic dynamical systems they define. We will focus especially on shift equivalence and strong shift equivalence, from several viewpoints, including polynomial matrices and algebraic K-theory. The lectures are designed such that the algebraic discussion can be of general interest, beyond application to symbolic dynamics.

In addition we have lectures on two related topics. We discuss inverse problems for nonnegative matrices, including the inverse spectral problem and generalizations, for matrices over the reals and other rings. We also consider the automorphism group of a shift of finite type, especially, its dimension representation. Ideas developed for the study of this group were fundamental to Wagoner's strong shift equivalence complex, and to the Kim-Roush counterexamples to Williams' Shift Equivalence Conjecture, which were constructed in the setting of this complex.

The lectures are too ambitious for complete proofs. Appendices contain some additional proof, attributions, references and remarks.

The lectures are listed below and then outlined. The final content might change a bit, as the lectures are not all completely written out as of the writing of this abstract.

Symbolic dynamics and the stable algebra of matrices

Four lectures by Boyle:

I. Basics.

II. Shift equivalence and strong shift equivalence over a ring.

III. Polynomial matrices.

IV. Inverse problems for nonnegative matrices.

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Four lectures by Schmieding:

V. A brief introduction to algebraic K-theory.

VI. The algebraic K-theoretic characterization of the refinement of strong shift equivalence over a ring by shift equivalence.

VII. Automorphisms of SFTs.

VIII. Wagoner's strong shift equivalence complex, and applications.

Outline of the lectures

I. Basics.

1. Topological dynamics.
2. Symbolic dynamics.
3. (Edge) shifts of finite type from matrices over $\mathbb{Z}_+$.
4. The continuous shift-commuting maps. Block codes. Higher block presentations. Matrix powers and SFT powers.
5. Periodic points and nonzero spectrum.
6. Classification of SFTs
7. Strong shift equivalence of matrices.
8. Shift equivalence of matrices.
9. Williams' Shift Equivalence Conjecture (1974). Kim-Roush Counterexamples. The gap.

II. Shift equivalence and strong shift equivalence over a ring.

1. Shift equivalence (SE) over $\mathbb{Z}_+$. Dynamical meaning. Reduction to shift equivalence over $\mathbb{Z}$.
2. Strong shift equivalence (SSE) over a ring.
3. Shift equivalence over a field, PID or $\mathbb{Z}$.

4. SE-$\mathbb{Z}$: example classes. Two eigenvalues. Taussky-Todd.
5. SE-$\mathbb{Z}$ in terms of direct limits.
6. SE-$\mathbb{Z}$ in terms of $\mathbb{Z}[t]$-modules.

III. Polynomial matrices.

1. Background.
Algebraic invariants for flow equivalence of SFTs.
Vertex SFTs from zero-one matrices. Advantages of definition from $\mathbb{Z}_+$ vs. zero-one matrices (edge SFTs vs. vertex SFTs): functoriality, conciseness, proof tools.

2. Presenting SFTs with polynomial matrices. Conciseness.
3. Algebraic invariants in the polynomial setting. Functoriality.
4. Classification of SFTs in the polyomial setting. Positive equivalence. Matrices with No Zero Cycles (NZC).
5. Proof tools in the polynomial setting.
6. SE, SSE, positive equivalence for other rings, other symbolic systems.
7. The table is set for the K-theory connection.

IV. Inverse problems for nonnegative matrices.

Perron-Frobenius theory.
Primitive/irreducible/general nonnegative matrices.
Analogue for SFT Classification.

The NNIEP (nonnegative matrix inverse eigenvalue problem). Background.
The inverse spectral problem for primitive matrices.
The nonzero inverse spectral problem.
Spectral Conjecture (Boyle-Handelman).
Theorems of Boyle-Handelman, Kim-Ormes-Roush and Laffey.
Role of polynomial matrices.
Generalized Spectral Conjectures (BH).

V. A brief introduction to algebraic K-theory.

1. Definition of K_1 of a ring, and some examples.
2. The group NK_1 of a ring.
3. The class group of nilpotent endomorphisms Nil_0, and its relationship with NK_1.
4. Some examples, and properties of the group NK_1
5. Begin describing the connection between these K-theoretic objects and (strong) shift equivalence over a ring.

VI. The algebraic K-theoretic characterization of the refinement of strong shift equivalence over a ring by shift equivalence.

With applications to symbolic dyamical systems.
1. (Strong) shift equivalence over a ring, and the general algebraic shift equivalence problem.
2. The connection between (strong) shift equivalence and the groups NK_1, Nil_0.
3. Elementary equivalence, the algebraic description of strong shift equivalence.
4. The refinement of shift equivalence by strong shift equivalence over a ring, and its connection to NK_1, Nil_0.

VII. Automorphisms of SFTs.

1. Definition of the automorphism group of a shift of finite type, some examples.

VIII. Wagoner's strong shift equivalence complex, and applications.