Final Exam–Stat 100–Fall 1997

You may use calculators, and ONE page of notes (writing allowed both sides). Each problem is worth 20 points. Do not spend too much time on any one problem. Put a box around the final answer to a question. EACH PROBLEM SHOULD BE ANSWERED ON A SEPARATE PAGE. Use the back side of a page if necessary.

1. Let A and B be two events. Suppose the probability that A occurs is .3, the probability that B occurs is .4, and the probability that at least one of A or B occurs is .5.
 a. (7 pts.) What is the probability that B does not occur?
 b. (7 pts.) What is the probability that both A and B occur?
 c. (6 pts.) Are the events A and B independent? Justify your answer.

2. A jar contains 10 white balls and 6 red balls. A ball will be chosen at random and removed from the jar. Then a second ball will be chosen at random from the jar.
 a. (7 pts.) What is the probability that the first ball chosen is red?
 b. (7 pts.) Given that the first ball chosen is red, what is the probability that the second ball chosen is red?
 c. (6 pts.) What is the probability that both balls chosen will be red?

3. A retailer sells three brands of barbecue unit (A,B and C). The profit on brand A is $30 per unit, the profit on brand B is $50 per unit, and the profit on brand C is $50 per unit. Of the units sold, 40% are brand A, 25% are brand B and 35% are brand C.
 Let X represent the profit on a sale.
 a. (5 pts.) What is the probability distribution for X?
 b. (5 pts.) What is $E(X)$?
 c. (5 pts.) What is the standard deviation of X?
 d. (5 pts.) What is the retailer's expected profit from the next 9 units sold?

4. Consider a population of 10,000 physicians of whom 4000 favor aspirin brand A. Draw a random sample without replacement of $n = 50$ physicians, and record the number X of physicians who favor aspirin brand A.
 a. (4 pts.) Explain why it is a reasonable approximation to assume that X has a binomial distribution.
 b. (10 pts.) Approximately what is $P(20 \leq X \leq 30)$?
 c. (6 pts.) If the sample size is only $n = 15$, then approximately what is $P(X < 9)$?

5. Over the past several years, the mean score on the final of a statistics course has been 130 points out of 200, with a standard deviation of 40 points. This year, there are three sections of the course, with a total of 60 students.
 Based on the data above, what is the probability that the average (mean) score of these 60 students will be above 125?

************THERE ARE MORE PROBLEMS ON THE BACK SIDE************

6. The National Assessment of Educational Progress (NEAP) tests 17-year-old students nationwide on a variety of subjects. In 1992, a random sample of 1,000 students in second-year algebra averaged 307 on the mathematics test, with a standard deviation of 100.

Find a 95% confidence interval for the national average of second-year algebra students on the test.

7. Measurements of the acidity (pH) of 9 rain samples have a mean of 4.2 and a standard deviation of 0.4.

a. (16 points) Test $H_0 : \mu = 4.4$ versus $H_1 : \mu \neq 4.4$ at $\alpha = 0.05$.
b. (4 points) Estimate the P-value.

8. To find out whether a new LDP-lowering drug is effective, 5 patients are selected to be given the drug and 6 are selected as the control group. The average LDP level for the treatment group and control group are 89.3 and 105.0, respectively, and the standard deviations are 15.5 and 20.7.

a. (15 points) Construct a 95% confidence interval for the difference of the mean LDP levels.
b. (5 points) What assumptions justify your procedure?

9. A small pilot study is made of a new drug (A) intended to lower blood cholesterol levels. Five pairs of men with similar initial cholesterol levels are chosen for the study. From each pair, one is randomly chosen to take the new drug A for a month, while the other takes a standard drug B. At the end of the month, the following decreases in cholesterol level are recorded (a negative number reflects an increase in cholesterol level).

drug A	40	30	0	-10	20
drug B	20	20	0	-15	5

a. (16 points) Make an appropriate test of the claim that the new drug is more effective than the standard drug, at significance level $\alpha = .05$. Clearly state the null and alternate hypotheses, test statistic, rejection region and conclusion.

b. (4 points) State any assumptions you make to justify your procedure. Is the central limit theorem a part of what justifies this procedure? Briefly explain why or why not.

10. A group of 100 men with severe depression is split into two groups of 50. One group is treated with a certain drug, while the other group is put on an exercise regime. After three months, assessment of depression in the groups gives the following data.

	Better	Same	Worse	Total
Drug	30	12	8	50
Exercise	15	30	5	50
Total	45	42	13	100

a. (18 pts.) Test the claim that the two treatments have different effects, at $\alpha = .01$.
b. (2 pts.) What rule-of-thumb criterion justifies the use of the test you used?