

Math 130 – Spring 2015 – Final Exam

- NO CALCULATORS OR ELECTRONIC DEVICES ALLOWED.
- Use a separate answer sheet for each question.
- Give your pledge on page 1 only, covering the whole test.
- Draw a box around a final answer to a problem.

1. (15 points)

(a) (6 pts) You are given the following table of values:

x	1	2	3	4
$f(x)$	2	4	1	3
$f'(x)$	-6	-7	-8	-9
$g(x)$	2	3	4	1
$g'(x)$	2/7	3/7	4/7	5/7

Let $h(x) = g(f(x))$. What is the linear approximation to $h(3.02)$?

(This is also called the differential or tangent line approximation.) Do not simplify your answer.

(b) (14pts) Suppose for a certain kind of rectangular frame, the cost of the frame is \$10 per foot for the vertical sides and \$20 per foot for the horizontal sides. What is the largest area the frame could enclose at a cost of \$160 ?

2. (15 points)

(a) Compute the following limits. Possible correct answers are a number, ∞ , $-\infty$ or DNE (“does not exist”). (3 pts each)

$$(i) \lim_{x \rightarrow \infty} \frac{x^2 \sin(x)}{e^x}$$

$$(ii) \lim_{x \rightarrow 2^+} \frac{x^2 - 2x}{x^2 - 4}$$

$$(iii) \lim_{x \rightarrow \infty} \frac{x \ln(x)}{x^2 + 1}$$

(b) (6 pts) Find the equation of the tangent line to the curve $y = (\sin x)/x$ at the point where $x = \pi$.

3. (18 points) Evaluate the following definite integrals. (6 pts. each)

$$(a) \int_1^2 \frac{x^2 + \sqrt{x}}{x} dx$$

$$(b) \int_1^4 \frac{e^{-1/x}}{x^2} dx$$

$$(c) \int_{x=0}^{\pi/4} \frac{\cos x}{8 - \sqrt{2} \sin x} dx$$

4. (15 points)

(a) Suppose the rate of infection of a certain disease (in units of people per month) is modeled over a period of six months by the function $f(t) = 400(6t - t^2)$, where t is the time (in months) after the disease breaks out. In this model,

(i) (8 pts) What is the total number of people who are infected with the disease over the first six months after it breaks out?

(ii) (4 pts) At what time t is the rate of infection greatest?

(b) (3 pts) A certain number Q is approximated by sums of the form

$$\sum_{i=1}^n \sqrt{5 + i(3/n)} (3/n)$$

and these sums converge to Q as $n \rightarrow \infty$. Write a definite integral which is equal to Q . Do not compute the integral.

5. (16 points)

(a) (8 pts) A spherical shaped balloon is inflating at a rate of 2 cubic inches per second. How fast is the radius of the balloon increasing when the radius is 3 inches?

(b) (8 pts) A radioactive substance is decaying exponentially. At noon there is 90 grams. One hour later, there is 30 grams.

At what time t hours after noon will there be 5 grams of the substance?

6. (16 points) The function $f(x) = \frac{\ln(x)}{x}$ satisfies

$$f'(x) = \frac{1 - \ln(x)}{x^2} \quad \text{and} \quad f''(x) = \frac{-3 + 2 \ln(x)}{x^3}.$$

(i) (1 pt) What is the domain of f ?

(ii) (4 pts) Find all asymptotes of f .

(iii) (4 pts) On which intervals is f increasing, and on which intervals is f decreasing?

(iv) (4 pts) Determine the intervals on which the graph of f is concave up or down.

(v) (3 pts) Graph f .