A few sample problems for inferential statistics

Problems.

1. Suppose $X_1, \ldots, X_{100}$ are i.i.d random variables which have uniform distribution on $[a-2, a+2]$, where a is unknown. Suppose the random sample produces sample mean equal to 3.
Compute a 95% confidence interval for a.

2. In a mythical national survey, 225 students are randomly selected from those taking calculus, and asked if calculus is their favorite subject. 100 students reply that calcululs is their favorite subject. Give a 95% confidence interval for the proportion of all students taking calculus who consider it their favorite subject.

3. Suppose in a random sample of 225 undergraduate men at UMD that the average best (highest weight) bench press is 150 pounds, with sample standard deviation of 20 pounds. Compute a 95% confidence interval for the average best bench press for for UMD undergraduate men.

Solutions to the problems are on the following pages.

Solutions.

1. Suppose $X_1, \ldots, X_{100}$ are i.i.d random variables which have uniform distribution on $[a-2, a+2]$, where a is unknown. Suppose the random sample produces sample mean equal to 3.
Compute a 95% confidence interval for a.

1. SOLUTION

A random variable with uniform distribution on $[a-2, a+2]$ has mean $\mu = a$. So, a confidence interval for μ is a confidence interval for a. Because $n = 100$ is large, the confidence interval provided by the Central Limit Theorem applies:

$$\left(\overline{X} - 1.96\frac{\sigma}{\sqrt{n}},\ \overline{X} + 1.96\frac{\sigma}{\sqrt{n}}\right)$$

A random variable with uniform distribution on $[a-2, a+2]$ has standard deviation $\sigma = 4/\sqrt{12}$. Our sample mean is 3. Substituting, we get

$$\left(3 - (1.96)\frac{(4/\sqrt{12})}{\sqrt{100}},\ 3 + (1.96)\frac{(4/\sqrt{12})}{\sqrt{100}}\right)$$
$$= (2.73,\ 3.27)\ .$$

2. In a mythical national survey, 225 students are randomly selected from those taking calculus, and asked if calculus is their favorite subject. 100 students reply that calcululs is their favorite subject. Give a 95% confidence interval for the proportion of all students taking calculus who consider it their favorite subject.

SOLUTION

We will plug into the 95%confidence interval formula for population proportion,

$$\left(\widehat{p} - 1.96\frac{\sqrt{\widehat{p}(1-\widehat{p})}}{\sqrt{n}},\ \widehat{p} + 1.96\frac{\sqrt{\widehat{p}(1-\widehat{p})}}{\sqrt{n}}\right)$$

Here $\widehat{p} = 100/225 = 20/45 = 4/9$ and $n = 225$, so the interval is

$$\begin{aligned} &= \left(4/9 - 1.96\frac{\sqrt{(4/9)(5/9)}}{\sqrt{225}},\ 4/9 + 1.96\frac{\sqrt{(4/9)(5/9)}}{\sqrt{225}}\right) \\ &= \left(4/9 - 1.96\frac{\sqrt{20}}{(9)(15)},\ 4/9 + 1.96\frac{\sqrt{20}}{(9)(15)}\right) \\ &\approx (.38,\ .51) \end{aligned}$$

3. Suppose in a random sample of 225 undergraduate men at UMD that the average best (highest weight) bench press is 150 pounds, with sample standard deviation of 20 pounds. Compute a 95% confidence interval for the average best bench press for for UMD undergraduate men.

SOLUTION

We use for the interval the formula

$$\left(\overline{X} - 1.96\frac{s}{\sqrt{n}},\ \overline{X} + 1.96\frac{s}{\sqrt{n}}\right)$$

Here the sample mean is 150 and $s = 20$. So the desired 95% confidence interval, in pounds, for the average best bench press of UMD undergraduate men is

$$\begin{aligned} &\left(150 - 1.96\frac{20}{\sqrt{225}},\ 150 + 1.96\frac{20}{\sqrt{225}}\right) \\ &= \left(150 - 1.96(.8),\ 150 + 1.96(.8)\right) \\ &= (48.4,\ 51.6)\ . \end{aligned}$$