Math 131 – Fall 2015 – Boyle –Exam 1

• NO CALCULATORS OR ELECTRONIC DEVICES ALLOWED.
• Use a separate answer sheet for each question; use the back side of an answer sheet if you need more space to answer a question.
• Give your pledge on page 1 only, covering the whole test.
• Draw a box around a final answer to a problem.

1. (10 points) What is the average value of $\sin x$ on the interval $[0, \pi]$?.

2. (13 points) Compute $\int_{x=1}^{3} x \ln x \, dx$.

3. (12 points) What is $\int_{x=1}^{\infty} \frac{1}{\sqrt{x}} \, dx$?
(The possible correct answers are a number, ∞, $-\infty$ or DNE (does not exist).)

4. (9 points) Graph the level curves in the xy plane for the function $z = x^2 + y^2$ for the values $z = 0$, $z = 1$ and $z = 4$. Put all the level curves on the same graph.

5. (16 points) Use the total differential (i.e., linear approximation) to estimate $\sqrt{(8.04)^2 + (5.98)^2}$. Choose appropriate numbers and do the arithmetic to give your final answer in decimal form

6. (14 points) Let $f(x, y) = x^2 + xy + y^2 - 6x - 3$.

(a) (4 pts) Find every critical point of f.

(b) (10 pts) At each critical point, determine whether f has a local minimum, a local maximum or a saddle.

7. (14 points) Let R be the region in the xy plane bounded between the graphs of $y = x^2$ and $y = x$.

(a) (4 pts) Draw those graphs and indicate in your picture what R is.

(b) (10 pts) Compute the integral $\int \int_R x^2 y \, dxdy$.

THERE IS ANOTHER QUESTION
ON THE BACK SIDE OF THIS PAGE.

8. (12 points) Let I denote a given definite integral $\int_{x=a}^{b} f(x)\,dx$. Let L_n, T_n, S_n denote the estimates of I by the Left Sum, Trapezoid Rule and Simpson's Rule (respectively), using the values of the function f at equally spaced points $x_0, x_1, \ldots, x_n$. There are constants C_1, C_2, C_3 (depending on f and $[a, b]$, but not on n) and constants k, m, p such that the following hold:
$|L_n - I| \le C_1(1/n^k)$; $|T_n - I| \le C_2(1/n^m)$; $|S_n - I| \le C_3(1/n^p)$.
(We call the right hand side of such an inequality an *error bound*.)

(a) (4 pts) What are k, m, p?

(b) (2 pts) Suppose E is the error bound at $n = 16$, and you want to use another n for which the error bound is $E/(10,000)$.
(i) What should the new n be for the L_n approximation?
(ii) What should the new n be for the S_n approximation?

(c) (2 pts) Suppose $[a, b] = [-2, 2]$ and $n = 4$.
What are the points x_0, x_1, x_2, x_3, x_4?

(d) (4 pts) Graph $f(x) = x^2$ over the interval $[-2, 2]$, and draw a shaded region whose area is the estimate T_4 for $\int_{x=-2}^{2} x^2\,dx$. Make your picture large and clear enough that we can see you understand.