

An example computation using matrix multiplication

Joe sells coffee and donuts in three cities (which I'll call cities 1,2 and 3).

A cup of coffee costs \$1.00 in city1, \$2.00 in city 2 and \$3.00 in city 3. A donut costs \$1.50 in city1, \$2.50 in city 2 and \$3.50 in city 3. This information is summarized in the 2×3 matrix

$$A = \begin{pmatrix} 1.00 & 2.00 & 3.00 \\ 1.50 & 2.50 & 3.50 \end{pmatrix} .$$

Yesterday, Joe sold 60, 70 and 80 cups of coffee in cities 1,2 and 3 (respectively) and he sold 52, 42 and 62 donuts in cities 1,2 and 3 (respectively). This is summarized in the 3×2 matrix

$$B = \begin{pmatrix} 60 & 52 \\ 70 & 42 \\ 80 & 62 \end{pmatrix}$$

Now we compute the matrix product

$$C = AB = \begin{pmatrix} 1.00 & 2.00 & 3.00 \\ 1.50 & 2.50 & 3.50 \end{pmatrix} \begin{pmatrix} 60 & 52 \\ 70 & 42 \\ 80 & 62 \end{pmatrix} = \begin{pmatrix} 440 & 324 \\ 545 & 402 \end{pmatrix}$$

For example, the computation for the entry $B(2,1)$ is

$$\begin{aligned} B(2,1) &= (1.50)(60) + (2.50)(70) + (3.50)(80) \\ &= 90 + 175 + 280 = 545 . \end{aligned}$$

We interpret this by remembering the units and “cancelling”. For example, $(1.50 \text{ \$ /cup})(60 \text{ cups}) = (1.50)(60) (\text{\$/cup})(\text{cup}) = (1.50)(60) \text{ \$}$. The number $B(2,1) = 545$ is the total number of dollars Joe collected yesterday selling coffee in the three cities. The other entries of B have similar interpretations.

Matrices are often used to organize calculations like this. Choosing and “cancelling ” units can be helpful for checking that the matrices are set up to do the computation you want.