MATH 131 — EXAM 1 SOLNS - SPRING 2016

1. $$\int_{x=0}^{\infty} e^{-kx}\,dx = \lim_{b\to\infty} \int_{x=0}^{b} e^{-kx}\,dx$$

$$= \lim_{b\to\infty} \left[-\frac{1}{k} e^{-kx}\right]_{x=0}^{b}$$

$$= \lim_{b\to\infty} \left(-\frac{1}{k} e^{-kb} - \left(-\frac{1}{k} e^{0}\right)\right) = \boxed{\frac{1}{k}}$$

2. $$\int_{x=0}^{\pi} \underbrace{x}_{u} \underbrace{\sin 3x}_{v'}\,dx = \Big[\underbrace{x}_{u} \underbrace{\left(-\tfrac{1}{3}\cos 3x\right)}_{v}\Big]_0^{\pi} - \int_0^{\pi} \underbrace{(1)}_{u'}\underbrace{\left(-\tfrac{1}{3}\cos 3x\right)}_{v}\,dx$$

$$= \pi\left(-\tfrac{1}{3}\cos(3\pi)\right) - 0 + \frac{1}{3}\int_0^{\pi} \cos 3x\,dx$$

$$= \frac{\pi}{3} + \frac{1}{3}\left[\frac{1}{3}\sin(3x)\right]_{x=0}^{\pi}$$

$$= \frac{\pi}{3} + \frac{1}{3}\left[0 - 0\right]$$

$$= \boxed{\pi/3}$$

③ (a) $\displaystyle\int_1^{\infty} \frac{1}{\sqrt[3]{x}}\,dx = \int_1^{\infty} x^{-1/3}\,dx$

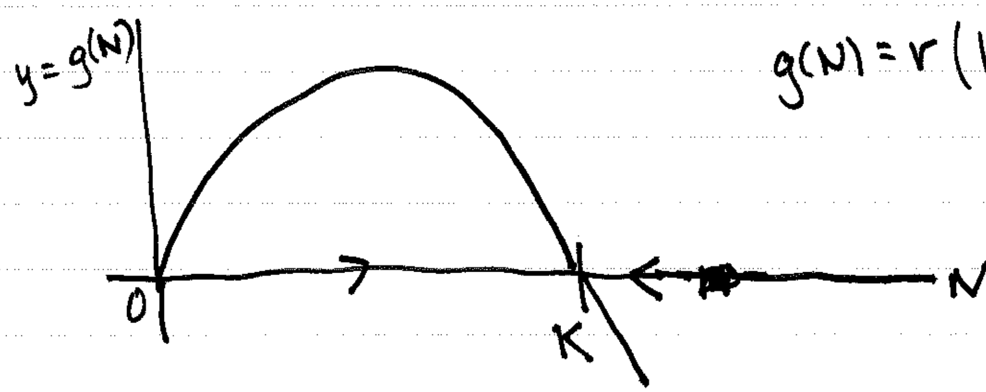
$\displaystyle = \lim_{b\to\infty} \int_1^b x^{-1/3}\,dx = \lim_{b\to\infty} \left[\frac{3}{2}x^{2/3}\right]_{x=1}^{b}$

$\displaystyle = \lim_{b\to\infty} \left(\frac{3}{2}b^{2/3} - \frac{3}{2}\right) = \boxed{+\infty}$

(b) $\displaystyle\int_{x=1}^{\infty} \frac{1}{x^p}\,dx < \infty$ if $\boxed{p>1}$

④ (a) $N=0$ and $N=K$

(b)



$g(N) = r\left(1-\frac{N}{K}\right)N$

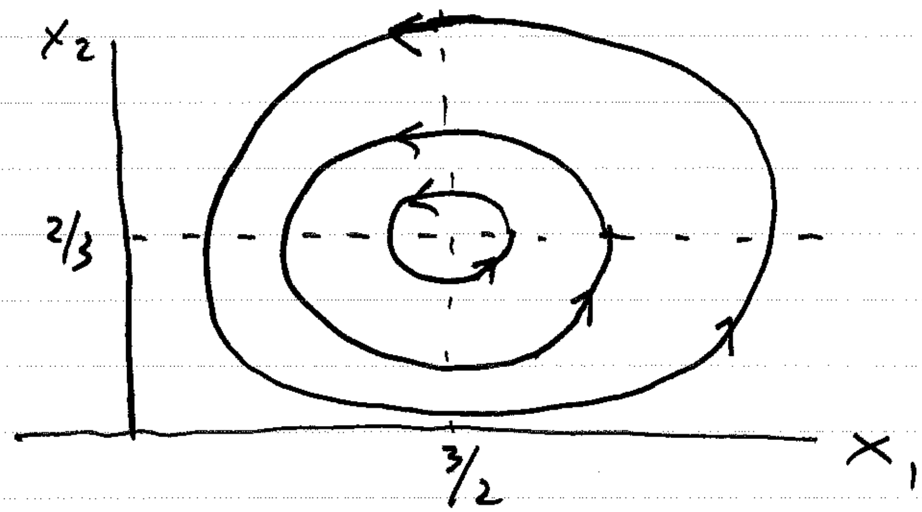
(c) $N=0$ is unstable.
$N=K$ is stable

(7) (a) eq. pts $(0,0)$ and $\left(\frac{3}{2}, \frac{2}{3}\right)$

$$\frac{dx_1}{dt} = x_1(2-3x_2)$$

$$\frac{dx_2}{dt} = x_2(-3+2x_1)$$

(b)



(c) $(0,0)$: unstable
$(3/2, 2/3)$: stable

(8) (a) first sphere center $= (9,3,1)$ radius $= 2$
second sphere center $= (6,6,-2)$ radius $= 3$

(b) distance between centers $= \sqrt{3^2+3^2+3^2} = \sqrt{27}$
sum of radii $= 2+3 = 5 < \sqrt{27}$
spheres do $\boxed{\text{not}}$ overlap

(5)

(a) $\dfrac{dC}{dt} = -k(C-r)$ → let $u = C - r$

$$\frac{du}{dt} = -ku$$

$$u = u_0 e^{-kt}$$

$$C - r = (C_0 - r)e^{-kt}$$

$$\boxed{C = r + (C_0 - r)e^{-kt}}$$

(b) $\displaystyle\lim_{t \to \infty} C(t) = \boxed{r}$

(6)

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\int y\,dy = \int x\,dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$$

$$y^2 = x^2 + \underbrace{C}_{\text{new } C}$$

$$y = \pm\sqrt{x^2 + C}$$

at $\cancel{x} = 0$: $-5 = \pm\sqrt{0^2 + C}$

So,

$$\boxed{y = -\sqrt{x^2 + 25}}$$