Math 240 Exam 2 Sample 1 **Spring 2008**

Please put problem 1 on answer sheet 1

1. (a) Find the determinant of

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

using cofactor expansion along row 2. Do not simplify.

(b) Find the area of the triangle with vertices $(0, 0)$, $(4, 1)$ and $(3, -2)$.

Please put problem 2 on answer sheet 2

2. (a) Use Cramer's Rule to find the solution to the system of equations given below. Note that your solutions will depend on α. Do not simplify.

$$\begin{aligned} 2x_1 - 3\alpha x_2 &= 7 \\ -x_1 + 5x_2 &= 1 \end{aligned}$$

(b) Let V be the vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$ and let $T : V \to \mathbb{R}$ be the linear transformation $T(f) = \int_{-1}^{1} f(x)\ dx$.

i. Find $T(x^2)$.

ii. Show that $f(x) = \sqrt[3]{x}$ is in Ker T.

Please put problem 3 on answer sheet 3

3. (a) Two of the following three are not vector spaces. Identify which two are not and justify why they are not.

i. $\left\{ \begin{bmatrix} a \\ b \end{bmatrix} \middle| \ a, b \in \mathbb{R},\ a \leq b \right\}$

ii. $\left\{ bt + ct^2 \ \middle| \ b, c \in \mathbb{R},\ bc \neq 0 \right\}$

iii. $\left\{ f : \mathbb{R} \to \mathbb{R} \ \middle| \ f(3) = 0 \right\}$

(b) Show that the set S of vectors of the form

$$\begin{bmatrix} s - 3t \\ t + s \\ 2s \\ t - 4s \end{bmatrix} \quad \text{with } s, t \in \mathbb{R}$$

is a subspace of $\mathbb{R}^4$ by finding a matrix A with $S = \text{Col}\ A$.

Turn Over!

Please put problem 4 on answer sheet 4

4. (a) Show that the set of functions $\{1+t^2, 3, 1-t^2\}$ is a linearly dependent subset of $\mathbb{P}_2$.

   (b) Given the matrix

$$A = \begin{bmatrix} 1 & 0 & 3 & 2 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 1 & 1 \end{bmatrix}$$

      i. Find a basis for Col A.

      ii. Find a basis for Nul A.

   (c) Suppose A is a 3×7 matrix. Answer the following *briefly*.

      i. What is the smallest that dim(Nul A) could be? Why?

      ii. What is the largest that dim(Nul A) could be? Why?

Please put problem 5 on answer sheet 5

5. Suppose $\mathcal{B} = \{t, 2t+1, t^2+t+1\}$ and $\mathcal{C} = \{1, t+1, t^2+t+1\}$ are both bases for P_2.

   (a) Suppose $[p(t)]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Find $p(t)$.

   (b) Find the change of basis matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$.