MATH 240 – Spring 2013 – Exam 3

CALCULATORS ARE <u>NOT</u> ALLOWED
CLOSED BOOK, NO NOTES
TURN OFF ALL ELECTRONIC DEVICES

There are 5 questions. Answer each on a separate sheet of paper. Use the back side if needed.

On each sheet, put your name, your section leader's name and your section meeting time.

When a question has a short final answer, put a BOX around that answer.

NOTATION: If A is a matrix, then A^T denotes the transpose of A.

1. (20 points) Let $A = \begin{pmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$. Find a diagonal matrix D and an invertible matrix P such that $P^{-1}AP = D$. (Do not compute P^{-1}.)

2. (21 points) Determine which of the following matrices are diagonalizable:

$$A = \begin{pmatrix} 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 3 \\ 4 & 2 & 0 & 1 \\ 5 & 3 & 1 & -5 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}.$$

Justify your answers.

3. (a) **(10 points)** Compute the area of the parallellogram P in $\mathbb{R}^3$ whose four corners are the points

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}.$$

(b) (5 points) Give an example of two matrices with the same characteristic polynomial which are not similar. No justification required.

THERE ARE MORE QUESTIONS ON THE OTHER SIDE OF THIS PAPER.

4. (a) **(5 points)** Let W be the span of the vectors $\begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 7 \\ 7 \\ 7 \\ 7 \\ 8 \end{pmatrix}$.

Write down a matrix A such that $Ax = 0$ if and only if x is in $W^{\perp}$. No justification necessary.

(b) **(5 points)** Let $P = \begin{pmatrix} .2 & .8 \\ .4 & .6 \end{pmatrix}$. As n goes to infinity, P^n approaches a matrix Q. What is Q?

(c) **(15 points)** For each statement below, write TRUE or FALSE. No justification necessary.

i. If W is a subspace of $\mathbb{R}^n$ and W is not the trivial space $\{0\}$, then W has an orthonormal basis.

ii. If $\mathbb{R}^n$ has an orthonormal basis of eigenvectors of a matrix A, then A is a symmetric matrix.

iii. The geometric multiplicity of an eigenvalue is less than or equal to its algebraic multiplicity.

iv. If A is a symmetric matrix, then A is an orthogonal matrix.

v. If x is the closest vector in the column space of A to b, then x is a least squares solution for the equation $Ax = b$.

5. In this problem A is an $m \times n$ matrix and b is a vector in $\mathbb{R}^m$.

(a) **(5 points)** There is an equation involving A^T whose solutions are precisely the least squares solutions to $Ax = b$. What is that equation?

(b) **(12 points)** Now consider the specific matrices

$$Q = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad R = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}.$$

Let $A = QR$. Find a least squares solution to $Ax = b$.

(c) **(4 points)** Is the least squares solution in part (b) unique? Briefly justify your answer.