Automorphisms of zero entropy dynamical systems

The symmetries of a dynamical system X form an interesting and often quite complicated group called its automorphism group. A celebrated result of Boyle, Lind, and Rudolph is that the automorphism group of even very easily described (but positive entropy) systems frequently contains isomorphic copies of all of the following (among many others): every finite group, the free group on two generators, and the direct sum of countably many copies of $\mathbb{Z}$. This rich subgroup structure makes it challenging to find groups that don't embed into $\mathrm{Aut}(X)$ or even to prove when two systems have non-isomorphic automorphism groups. By contrast, a number of strong algebraic results have been obtained in recent years for symbolic dynamical systems with zero entropy. In this talk I will discuss recent joint work with B. Kra in which we obtain strong restrictions on $\mathrm{Aut}(X)$ for subshifts of stretched exponential growth. I will also discuss how some of the techniques developed to study automorphisms can be used to study properties of ergodic measures on a low complexity subshift.