Structure and Computability of Multidimensional Shift Spaces

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Abstract

Multidimensional symbolic dynamics is, formally speaking, the study of expansive actions of the groups $\mathbb{Z}^d$ on subspaces of the Cantor space. The more concrete version of this is that one considers the set $S^{\mathbb{Z}^d}$ of *configurations*, infinite d-dimensional grids of symbols drawn from a finite alphabet S. The group $\mathbb{Z}^d$ acts on $S^{\mathbb{Z}^d}$ by translation, and *shift spaces* are defined to be its topologically closed translation-invariant subsets. The most concrete examples are *shifts of finite type*, defined by a finite set of forbidden patterns that must not occur anywhere in the configurations.

In this talk, I will discuss the effects of, and connections between, computability and structural properties in this context. In particular, I will consider the class of countable shifts of finite type, which have some interesting geometric properties. Usually, the relationship between the two notions is that a structural or dynamical property of a multidimensional shift space, like entropy, can be completely or almost completely characterized by a computability condition. In some cases, restricting the structure of a shift space in some way may also reduce its computational complexity, and vice versa.