

**SUPPLEMENTARY TABLES FOR THE PAPER
“JOINTLY PERIODIC POINTS IN CELLULAR AUTOMATA:
COMPUTER EXPLORATIONS AND CONJECTURES”**

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These are tables supplementary to our Experimental Mathematics paper “Jointly periodic points in cellular automata: computer explorations and conjectures”. The tables are explained in that paper.

The tables are organized in four groups:

- Tables of some span 4 and 5 c.a. (Tables 1-2)
- FDense Tables (Tables 3-7)
- FPeriod Tables (Tables 8-24)
- FProbPeriod Tables (Tables 25-33)

In the tables, decimal output data are approximated by truncation; e.g., 1.429 becomes 1.42 rather than 1.43.

1. TABLES OF SOME SPAN 4 AND 5 C.A.

| Map | Tabular rule | Map | Tabular rule |
|-----|---------------------|-----|---------------------|
| 1 | 0000 1111 0010 1101 | 17 | 0011 1001 1100 1100 |
| 2 | 0000 1111 0100 1011 | 18 | 0011 1010 0011 1100 |
| 3 | 0001 1100 0011 1110 | 19 | 0011 1010 1100 0011 |
| 4 | 0001 1110 0101 1010 | 20 | 0011 1100 0101 0011 |
| 5 | 0010 1001 0110 1101 | 21 | 0011 1100 0101 1100 |
| 6 | 0010 1101 0000 1111 | 22 | 0011 1100 1010 0011 |
| 7 | 0011 0011 0110 0011 | 23 | 0011 1100 1010 1100 |
| 8 | 0011 0011 0110 1100 | 24 | 0011 1110 0001 1100 |
| 9 | 0011 0011 1001 0011 | 25 | 0100 1001 0110 1011 |
| 10 | 0011 0011 1001 1100 | 26 | 0100 1011 0000 1111 |
| 11 | 0011 0101 0011 1100 | 27 | 0101 1010 0001 1110 |
| 12 | 0011 0101 1100 0011 | 28 | 0101 1010 0111 1000 |
| 13 | 0011 0110 0011 0011 | 29 | 0110 1011 0100 1001 |
| 14 | 0011 0110 1100 1100 | 30 | 0110 1101 0010 1001 |
| 15 | 0011 1000 0111 1100 | 31 | 0111 1000 0101 1010 |
| 16 | 0011 1001 0011 0011 | 32 | 0111 1100 0011 1000 |

TABLE 1. The 32 span 4 onto c.a. of the 2 shift which fix $\dots 000\dots$ and are not linear in an end variable [Hedlund-Appel-Welch1963, Table I]. Maps 2, 6, 7 and 16 are one-to-one. The rule above for map 30 corrects a misprint in [Hedlund-Appel-Welch1963, Table I].

| Map | Tabular rule | Map | Tabular rule |
|-----|---|-----|---|
| 1 | 0001 0111 1110 1000 0001 0111 1111 0000 | 14 | 0100 1101 1111 0000 0100 1101 1011 0010 |
| 2 | 0001 1011 0111 0100 1110 0100 1111 0000 | 15 | 0110 0001 1010 1011 0110 0001 0110 0111 |
| 3 | 0010 0010 1111 0011 0010 1110 0000 1111 | 16 | 0110 1000 0111 1001 0110 0001 1110 1001 |
| 4 | 0010 1001 0110 1101 0100 1001 0110 1011 | 17 | 0110 1011 1100 0010 0100 1011 0001 1101 |
| 5 | 0010 1110 0000 1111 0010 1110 1111 0000 | 18 | 0111 0001 1011 0010 0111 0001 1000 1110 |
| 6 | 0100 0111 0001 0111 1011 1000 0000 1111 | 19 | 0111 0010 1011 0100 0111 0010 0111 1000 |
| 7 | 0100 0111 0100 1011 1000 1011 0100 1011 | 20 | 0111 1000 0100 1011 0111 1000 0111 1000 |
| 8 | 0100 1011 1000 0111 0100 1011 0100 1011 | 21 | 0111 1000 0100 1011 0111 1000 1011 0100 |
| 9 | 0100 1101 1011 0010 1000 1110 1011 0010 | 22 | 0111 1000 0100 1011 0111 1000 1111 0000 |
| 10 | 0100 1101 1011 0010 1100 1100 1011 0010 | 23 | 0111 1000 0100 1101 0111 1000 1000 1110 |
| 11 | 0100 1101 1101 0010 0011 0011 1101 0010 | 24 | 0111 1011 1000 0100 0100 1011 0000 1111 |
| 12 | 0100 1101 1101 0010 0111 0001 1101 0010 | 25 | 0111 1011 1100 0000 0100 1011 0000 1111 |
| 13 | 0100 1101 1101 0010 1111 0000 1101 0010 | 26 | 0111 1011 1100 0000 0100 1011 0100 1011 |

TABLE 2. 26 irregular span 5 onto maps of the 2 shift which fix $\dots 000\dots$ and are not linear in an end variable [Hedlund-Appel-Welch1963, Table XII].

2. FDENSE TABLES

| Map | 10-dense at | 13-dense at | Map | 10-dense at | 13-dense at |
|-----|-------------|-------------|-----|-------------|-------------|
| 1 | 11-24 | 13-24 | 17 | 17,18,20-24 | 24 |
| 2 | 10-24 | 13-24 | 18 | 17,19-24 | 23-24 |
| 3 | 18-24 | 24 | 19 | 19-24 | 23-24 |
| 4 | 21-24 | (27) | 20 | 17,19-23 | (25) |
| 5 | 17,19-23 | (25) | 21 | 19-24 | 23-24 |
| 6 | 10-24 | 13-24 | 22 | 19-24 | 23-24 |
| 7 | 10-24 | 13-24 | 23 | 19-24 | 21-22 |
| 8 | 21-24 | (27) | 24 | 17,19-24 | 23-24 |
| 9 | 11-24 | 13-24 | 25 | 17,19-23 | (25) |
| 10 | 19,21-24 | 24 | 26 | 11-24 | 13-24 |
| 11 | 18-24 | 24 | 27 | 19, 21-24 | 24 |
| 12 | 17,19-23 | (25) | 28 | 22-24 | (25) |
| 13 | 11-24 | 13-24 | 29 | 19-24 | 23-24 |
| 14 | 22-24 | (25) | 30 | 19-24 | 23-24 |
| 15 | 19-24 | 23-24 | 31 | 17,18,20-24 | 24 |
| 16 | 10-24 | 13-24 | 32 | 19-24 | 21-22 |

TABLE 3. The map numbers refer to the 32 span 4 maps of Table 1. Table 3 shows for the given sample of maps, and for $m = 10$ and $m = 13$, for which k in the range $[m, 24]$ the jointly periodic points in $P_k(S_2)$ are m -dense. If the map is not m -dense for any k in this range, then the number listed in parentheses is the smallest k for which the jointly periodic points in $P_k(S_2)$ are m -dense.

| Map | 10-dense at | 13-dense at | Map | 10-dense at | 13-dense at |
|--------------|-------------|-------------|--------------|-------------|-------------|
| F \circ 1 | 11-24 | 13,15-24 | F \circ 17 | 19,21-24 | 24 |
| F \circ 2 | 11-24 | 13-24 | F \circ 18 | 19-24 | 23-24 |
| F \circ 3 | 19-24 | 21-22 | F \circ 19 | 19-24 | (27) |
| F \circ 4 | 22-24 | (25) | F \circ 20 | 21-24 | 23 |
| F \circ 5 | 21-24 | 23 | F \circ 21 | 17,19-24 | 23-24 |
| F \circ 6 | 10-24 | 13-24 | F \circ 22 | 19-24 | (27) |
| F \circ 7 | 20-24 | 13-24 | F \circ 23 | 18-24 | 24 |
| F \circ 8 | 22-24 | (25) | F \circ 24 | 19-24 | 23-24 |
| F \circ 9 | 11-24 | 13,15-24 | F \circ 25 | 21-24 | 23 |
| F \circ 10 | 17-18,21-24 | 24 | F \circ 26 | 11-24 | 13,15-24 |
| F \circ 11 | 19-24 | 21,22 | F \circ 27 | 17-18,20-24 | 24 |
| F \circ 12 | 21-24 | 23 | F \circ 28 | 21-24 | (27) |
| F \circ 13 | 11-24 | 13,15-24 | F \circ 29 | 19-24 | (27) |
| F \circ 14 | 21-24 | (27) | F \circ 30 | 19-24 | (27) |
| F \circ 15 | 17-19,20-24 | 23-24 | F \circ 31 | 19,21-24 | 24 |
| F \circ 16 | 10-24 | 13-24 | F \circ 32 | 18-24 | 24 |

TABLE 4. The map numbers refer to the 32 span 4 maps of Table 1. F is the involution $F = x_0 + 1$. Table 4 shows for the given sample of maps, and for $m = 10$ and $m = 13$, for which k in the range $[m, 24]$ the jointly periodic points in $P_k(S_2)$ m -dense. If the map is not m -dense for any k in this range, then the number listed in parentheses is the smallest k for which the jointly periodic points in $P_k(S_2)$ are m -dense.

| Map | 10-dense at | Map | 10-dense at | Map | 10-dense at | Map | 10-dense at |
|------|-------------|------|-------------|------|-------------|------|-------------|
| F◦1 | 11-24 | F◦17 | 19,21-24 | D◦1 | 20-24 | D◦17 | 21-24 |
| F◦2 | 11-24 | F◦18 | 19-24 | D◦2 | 19,22-24 | D◦18 | 21-24 |
| F◦3 | 19-24 | F◦19 | 19-24 | D◦3 | 20-24 | D◦19 | 22-24 |
| F◦4 | 22-24 | F◦20 | 21-24 | D◦4 | 21,23-24 | D◦20 | 20,24 |
| F◦5 | 21-24 | F◦21 | 17,19-24 | D◦5 | 20,24 | D◦21 | 21,23-24 |
| F◦6 | 10-24 | F◦22 | 19-24 | D◦6 | 20,22,24 | D◦22 | 24 |
| F◦7 | 20-24 | F◦23 | 18-24 | D◦7 | 19,22-24 | D◦23 | 21-24 |
| F◦8 | 22-24 | F◦24 | 19-24 | D◦8 | 21,23-24 | D◦24 | 21-24 |
| F◦9 | 11-24 | F◦25 | 21-24 | D◦9 | 22-24 | D◦25 | 22-24 |
| F◦10 | 17-18,21-24 | F◦26 | 11-24 | D◦10 | 21,23-24 | D◦26 | 22-24 |
| F◦11 | 19-24 | F◦27 | 17-18,20-24 | D◦11 | 22-24 | D◦27 | 21,23-24 |
| F◦12 | 21-24 | F◦28 | 21-24 | D◦12 | 20-21,23-24 | D◦28 | 22-24 |
| F◦13 | 11-24 | F◦29 | 19-24 | D◦13 | 20-24 | D◦29 | 22-24 |
| F◦14 | 21-24 | F◦30 | 19-24 | D◦14 | 22-24 | D◦30 | 24 |
| F◦15 | 17-19,20-24 | F◦31 | 19,21-24 | D◦15 | 21,23-24 | D◦31 | 21-24 |
| F◦16 | 10-24 | F◦32 | 18-24 | D◦16 | 20,22,24 | D◦32 | 21-24 |

TABLE 5. The c.a. listed are compositions, e.g. $D \circ j$ is map j followed by D . The map numbers j refer to the 32 span 4 maps of Table 1. The map D is given by $x_0 + x_1$. The map F is the flip involution $F = 1 + x_0$. The data on $F \circ j$ are copied in from Table 4 for contrast with $D \circ j$.

| j | 10-dense at | j | 10-dense at | j | 10-dense at | j | 10-dense at |
|---|-------------|----|-------------|----|-------------|----|-------------|
| 1 | 18-24 | 9 | 17-19,21-24 | 17 | 18-20,22-24 | 25 | 11,13-24 |
| 2 | 19,21-24 | 10 | 19-24 | 18 | 17,21-24 | 26 | 11,13-24 |
| 3 | 17-24 | 11 | 17,19-24 | 19 | 17-24 | | |
| 4 | 18-19,21-24 | 12 | 18-24 | 20 | 18-24 | | |
| 5 | 17-24 | 13 | 16-23 | 21 | 19-24 | | |
| 6 | 15-24 | 14 | 16-24 | 22 | 19-24 | | |
| 7 | 16-24 | 15 | 18-24 | 23 | 20-24 | | |
| 8 | 11,13-24 | 16 | 18-19,21-24 | 24 | 11,13-24 | | |

TABLE 6. The map numbers j refer to the 26 “irregular” span 5 maps of [Hedlund-Appel-Welch1963, Table XII], copied in Table 2.

| j | 10-dense at | j | 10-dense at | j | 10-dense at | j | 10-dense at |
|---|-------------|----|-------------|----|-------------|----|-------------|
| 1 | 20-24 | 9 | 22-23 | 17 | 21,24 | 25 | 20,23-24 |
| 2 | 18-24 | 10 | 21,24 | 18 | 22,24 | 26 | 19,22-24 |
| 3 | 20,22-24 | 11 | 21-24 | 19 | 23,24 | 27 | 21,24 |
| 4 | 21,23-24 | 12 | 21-24 | 20 | 22-24 | 28 | 21,23-24 |
| 5 | 21,23-24 | 13 | 21-24 | 21 | 22,24 | 29 | 20,22-23 |
| 6 | 19-24 | 14 | 21-24 | 22 | 23,24 | 30 | 19-24 |
| 7 | 20,22-24 | 15 | 21-24 | 23 | 21-24 | 31 | 21,24 |
| 8 | 21-24 | 16 | 20-21,23 | 24 | 21-24 | 32 | 20,22-24 |

TABLE 7. As in Table 5, the map numbers refer to the 32 span 4 maps of Table 1. For such a map j , let $p_j(x_0, x_1, x_2, x_3)$ be the polynomial such that $(jx)_0 = p_j(x_0, x_1, x_2, x_3)$. Then a row j of Table 7 refers to the map f_j such that $(f_jx)_0 = x_0 + p_j(x_1, x_2, x_3, x_4)$. Equivalently, $f_j = x_0 + (j \circ S_2)$. For the map f_j , all k in the range $[10, 24]$ at which f_j is 10-dense are listed.

By comparison to Table 5, we see that altering the rules j as we have produces maps which look more “random” in the sense that all the 10-density is being achieved at k on the order of 2×10 . Note that the maps in Table 7 are by construction left permutative, and for these the jointly periodic points are known to be dense [Boyle-Kitchens1999].

3. FPERIOD TABLES

Output obtained from the FPeriod Program, discussed in Section 6 (Section FPeriod) , is compiled in the Tables below. In Tables 8-16, for a c.a. f , the numbers given for a row k are computed with respect to $P_k(S_N)$, the set of points fixed by the shift S_N . Except for Tables 15 and 16, the number of symbols N is 2. For a given k , L denotes the maximum f -period of a point in $P_k(S_N)$; P denotes the number of points in $P_k(S_N)$ which are f -periodic (so, $P + \text{Not-P} = N^k$); and ν_k denotes $\nu_k(f, S_N) = P^{1/k}$. In some later tables, ν_k^o is used to denote the k th root of the number of points of *least* S_n -period k which are periodic for f . The preperiod of a point x is the smallest nonnegative integer j such that $f^j(x)$ is f -periodic.

| k | Fraction Periodic | ν_k | $L^{1/k}$ | P | L | Not-P | Average Period | Average Preperiod | Maximum Preperiod |
|-----|-------------------|---------|-----------|-----------|-------|-----------|----------------|-------------------|-------------------|
| 1 | 0.500000 | 1.00 | 1.00 | 1 | 1 | 1 | 1.00 | 0.50 | 1 |
| 2 | 0.250000 | 1.00 | 1.00 | 1 | 1 | 3 | 1.00 | 1.25 | 2 |
| 3 | 0.500000 | 1.58 | 1.44 | 4 | 3 | 4 | 2.50 | 0.50 | 1 |
| 4 | 0.062500 | 1.00 | 1.00 | 1 | 1 | 15 | 1.00 | 3.06 | 4 |
| 5 | 0.500000 | 1.74 | 1.71 | 16 | 15 | 16 | 14.12 | 0.50 | 1 |
| 6 | 0.250000 | 1.58 | 1.34 | 16 | 6 | 48 | 5.12 | 1.25 | 2 |
| 7 | 0.500000 | 1.81 | 1.32 | 64 | 7 | 64 | 6.91 | 0.50 | 1 |
| 8 | 0.003906 | 1.00 | 1.00 | 1 | 1 | 255 | 1.00 | 7.00 | 8 |
| 9 | 0.500000 | 1.85 | 1.58 | 256 | 63 | 256 | 62.05 | 0.50 | 1 |
| 10 | 0.250000 | 1.74 | 1.40 | 256 | 30 | 768 | 29.01 | 1.25 | 2 |
| 11 | 0.500000 | 1.87 | 1.69 | 1,024 | 341 | 1024 | 340.67 | 0.50 | 1 |
| 12 | 0.062500 | 1.58 | 1.23 | 256 | 12 | 3840 | 11.57 | 3.06 | 4 |
| 13 | 0.500000 | 1.89 | 1.67 | 4,096 | 819 | 4096 | 818.80 | 0.50 | 1 |
| 14 | 0.250000 | 1.81 | 1.20 | 4,096 | 14 | 12,288 | 13.89 | 1.25 | 2 |
| 15 | 0.500000 | 1.90 | 1.19 | 16,384 | 15 | 16,384 | 14.99 | 0.50 | 1 |
| 16 | 0.000015 | 1.00 | 1.00 | 1 | 1 | 65535 | 1.00 | 15.00 | 16 |
| 17 | 0.500000 | 1.92 | 1.38 | 65,536 | 255 | 65,536 | 254.33 | 0.50 | 1 |
| 18 | 0.250000 | 1.85 | 1.30 | 65,536 | 126 | 196,608 | 125.73 | 1.25 | 2 |
| 19 | 0.500000 | 1.92 | 1.62 | 262,144 | 9,709 | 262,144 | 9708.96 | 0.50 | 1 |
| 20 | 0.062500 | 1.74 | 1.22 | 65,536 | 60 | 983,040 | 59.88 | 3.06 | 4 |
| 21 | 0.500000 | 1.93 | 1.21 | 1,048,576 | 63 | 1,048,576 | 62.99 | 0.50 | 1 |
| 22 | 0.250000 | 1.87 | 1.34 | 1,048,576 | 682 | 3,145,728 | 681.67 | 1.25 | 2 |
| 23 | 0.500000 | 1.94 | 1.39 | 4,194,304 | 2,047 | 4,194,304 | 2047.00 | 0.50 | 1 |

TABLE 8. The c.a. is $A = x_0 + x_1$ on the 2-shift: a linear, two-to-one map. If $k = qe$ with q odd and e a nonnegative integer power of 2, then there are exactly $(2^{eq}/2^e) = 2^{e(q-1)}$ points in $P_k(S_2)$ which are A -periodic, and $\nu_k = 2^{(q-1)/q}$. In particular, $\nu_k = 2^{(k-1)/k}$ if k is odd.

| k | Fraction Periodic | ν_k | $L^{1/k}$ | P | L | Not-P | Average Period | Average Preperiod | Maximum Preperiod |
|-----|----------------------|---------|-----------|--------|--------|-----------|-------------------|----------------------|----------------------|
| 1 | .5000 | 1.00 | 1.00 | 1 | 1 | 1 | 1.00 | 0.50 | 1 |
| 2 | .2500 | 1.00 | 1.00 | 1 | 1 | 3 | 1.00 | 1.25 | 2 |
| 3 | .5000 | 1.58 | 1.44 | 4 | 3 | 4 | 2.50 | 0.50 | 1 |
| 4 | .3125 | 1.49 | 1.41 | 5 | 4 | 11 | 2.50 | 1.31 | 3 |
| 5 | .3437 | 1.61 | 1.58 | 11 | 10 | 21 | 9.44 | 0.97 | 2 |
| 6 | .4375 | 1.74 | 1.61 | 28 | 18 | 36 | 11.31 | 0.69 | 2 |
| 7 | .0625 | 1.34 | 1.32 | 8 | 7 | 120 | 6.91 | 4.00 | 7 |
| 8 | .0195 | 1.22 | 1.18 | 5 | 4 | 251 | 3.25 | 6.58 | 12 |
| 9 | .1484 | 1.61 | 1.58 | 76 | 63 | 436 | 58.26 | 3.17 | 7 |
| 10 | .0888 | 1.57 | 1.52 | 91 | 70 | 933 | 18.17 | 7.77 | 17 |
| 11 | .0703 | 1.57 | 1.46 | 144 | 66 | 1,904 | 65.35 | 5.52 | 14 |
| 12 | .0576 | 1.57 | 1.36 | 236 | 42 | 3,860 | 24.44 | 10.98 | 34 |
| 13 | .0350 | 1.54 | 1.53 | 287 | 273 | 7,905 | 217.65 | 11.93 | 29 |
| 14 | .0201 | 1.51 | 1.39 | 330 | 105 | 16,054 | 12.65 | 36.60 | 74 |
| 15 | .0123 | 1.49 | 1.44 | 404 | 255 | 32,364 | 179.68 | 35.36 | 91 |
| 16 | .0232 | 1.58 | 1.54 | 1,525 | 1,008 | 64,011 | 272.23 | 33.28 | 98 |
| 17 | .0286 | 1.62 | 1.52 | 3,758 | 1,377 | 127,314 | 913.23 | 31.04 | 114 |
| 18 | .0091 | 1.54 | 1.53 | 2,386 | 2,250 | 259,758 | 2,026.85 | 55.23 | 152 |
| 19 | .0039 | 1.49 | 1.47 | 2,091 | 1,672 | 522,197 | 1,658.11 | 91.44 | 251 |
| 20 | .0015 | 1.44 | 1.31 | 1,635 | 240 | 1,046,941 | 14.16 | 279.12 | 575 |
| 21 | .0046 | 1.54 | 1.48 | 9,650 | 4,326 | 2,087,502 | 461.24 | 244.11 | 638 |
| 22 | .0011 | 1.47 | 1.40 | 4,896 | 1,848 | 4,189,408 | 1,158.45 | 274.42 | 647 |
| 23 | .0027 | 1.54 | 1.53 | 23,461 | 19,297 | 8,365,147 | 18,849.71 | 269.70 | 824 |

TABLE 9. The c.a. is J , the composition $x_0 + x_1$ followed by the involution $U = x_0 + x_{-2}x_1x_2 + x_{-2}x_{-1}x_1x_2$. This invertible c.a. U is the involution of the 2-shift which replaces x_0 with $x_0 + 1$ when $x[-2, 2] = 10x_011$. J is biclosing but not permutative.

| k | Fraction Periodic | ν_k | $L^{1/k}$ | P | L | Not-P | Average Period | Average Preperiod | Maximum Preperiod |
|-----|-------------------|---------|-----------|-------|-------|---------|----------------|-------------------|-------------------|
| 1 | 0.5000 | 1.00 | 1.00 | 1 | 1 | 1 | 1.00 | 0.50 | 1 |
| 2 | 0.2500 | 1.00 | 1.00 | 1 | 1 | 3 | 1.00 | 0.75 | 1 |
| 3 | 0.1250 | 1.00 | 1.00 | 1 | 1 | 7 | 1.00 | 1.62 | 2 |
| 4 | 0.3125 | 1.49 | 1.41 | 5 | 4 | 11 | 2.50 | 0.94 | 2 |
| 5 | 0.3437 | 1.61 | 1.58 | 11 | 10 | 21 | 9.44 | 0.66 | 1 |
| 6 | 0.0156 | 1.00 | 1.00 | 1 | 1 | 63 | 1.00 | 3.52 | 5 |
| 7 | 0.2812 | 1.66 | 1.54 | 36 | 21 | 92 | 17.62 | 1.05 | 2 |
| 8 | 0.0195 | 1.22 | 1.18 | 5 | 4 | 251 | 3.91 | 5.65 | 11 |
| 9 | 0.0546 | 1.44 | 1.22 | 28 | 6 | 484 | 5.82 | 5.18 | 11 |
| 10 | 0.1767 | 1.68 | 1.46 | 181 | 45 | 843 | 29.75 | 2.14 | 7 |
| 11 | 0.0703 | 1.57 | 1.55 | 144 | 132 | 1904 | 98.08 | 6.76 | 19 |
| 12 | 0.0012 | 1.14 | 1.12 | 5 | 4 | 4091 | 1.01 | 19.25 | 36 |
| 13 | 0.0556 | 1.60 | 1.49 | 456 | 182 | 7736 | 162.94 | 18.54 | 49 |
| 14 | 0.0261 | 1.54 | 1.35 | 428 | 70 | 15956 | 28.54 | 18.35 | 55 |
| 15 | 0.0342 | 1.59 | 1.45 | 1121 | 285 | 31647 | 138.58 | 21.60 | 58 |
| 16 | 0.0074 | 1.47 | 1.47 | 485 | 480 | 65051 | 430.96 | 71.09 | 146 |
| 17 | 0.0160 | 1.56 | 1.55 | 2109 | 1734 | 128963 | 1633.83 | 51.36 | 169 |
| 18 | 0.0060 | 1.50 | 1.41 | 1594 | 549 | 260550 | 334.44 | 70.40 | 233 |
| 19 | 0.0046 | 1.50 | 1.45 | 2452 | 1197 | 521836 | 834.45 | 92.00 | 227 |
| 20 | 0.0058 | 1.54 | 1.50 | 6165 | 3640 | 1042411 | 2700.37 | 70.21 | 211 |
| 21 | 0.0017 | 1.47 | 1.36 | 3627 | 693 | 2093525 | 585.86 | 356.39 | 817 |
| 22 | 0.0033 | 1.54 | 1.46 | 14004 | 4147 | 4180300 | 3305.59 | 251.62 | 864 |
| 23 | 0.0022 | 1.53 | 1.53 | 18746 | 18538 | 8369862 | 18491.96 | 262.30 | 900 |

TABLE 10. The c.a. E is the composition $x_0 + x_1$ followed by $x_0 + x_1x_2$, a linear 2-to-1 map followed by a degree 1 left permutative map.

| k | Fraction Periodic | ν_k | $L^{1/k}$ | P | L | Not-P | Average Period | Average Preperiod | Maximum Preperiod |
|-----|----------------------|---------|-----------|-------|-------|-----------|-------------------|----------------------|----------------------|
| 1 | 1.0000 | 2.00 | 1.00 | 2 | 1 | 0 | 1.00 | 0.00 | 0 |
| 2 | .5000 | 1.41 | 1.00 | 2 | 1 | 2 | 1.00 | 0.50 | 1 |
| 3 | .2500 | 1.25 | 1.00 | 2 | 1 | 6 | 1.00 | 1.12 | 2 |
| 4 | .1250 | 1.18 | 1.00 | 2 | 1 | 14 | 1.00 | 1.62 | 3 |
| 5 | .2187 | 1.47 | 1.37 | 7 | 5 | 25 | 1.62 | 2.03 | 4 |
| 6 | .4062 | 1.72 | 1.51 | 26 | 12 | 38 | 4.56 | 1.20 | 3 |
| 7 | .0703 | 1.36 | 1.32 | 9 | 7 | 119 | 6.91 | 4.65 | 9 |
| 8 | .0703 | 1.43 | 1.41 | 18 | 16 | 238 | 1.94 | 6.98 | 12 |
| 9 | .1796 | 1.65 | 1.48 | 92 | 36 | 420 | 15.52 | 2.55 | 7 |
| 10 | .0263 | 1.39 | 1.17 | 27 | 5 | 997 | 1.07 | 9.08 | 15 |
| 11 | .1782 | 1.70 | 1.53 | 365 | 110 | 1,683 | 77.16 | 3.79 | 16 |
| 12 | .0122 | 1.38 | 1.30 | 50 | 24 | 4,046 | 17.51 | 10.57 | 26 |
| 13 | .1049 | 1.68 | 1.53 | 860 | 260 | 7,332 | 199.90 | 6.20 | 21 |
| 14 | .0056 | 1.38 | 1.37 | 93 | 84 | 16,291 | 70.69 | 22.86 | 48 |
| 15 | .0340 | 1.59 | 1.43 | 1,117 | 225 | 31,651 | 117.64 | 13.52 | 42 |
| 16 | .0154 | 1.54 | 1.40 | 1,010 | 224 | 64,526 | 111.24 | 27.58 | 68 |
| 17 | .0135 | 1.55 | 1.45 | 1,770 | 612 | 129,302 | 558.46 | 41.02 | 112 |
| 18 | .0037 | 1.46 | 1.33 | 980 | 180 | 261,164 | 52.93 | 32.45 | 107 |
| 19 | .0078 | 1.54 | 1.50 | 4,125 | 2,242 | 520,163 | 824.24 | 52.35 | 168 |
| 20 | .0011 | 1.42 | 1.32 | 1,227 | 280 | 1,047,349 | 88.00 | 77.69 | 196 |
| 21 | .0008 | 1.42 | 1.39 | 1,731 | 1,092 | 2,095,421 | 29.02 | 180.81 | 480 |
| 22 | .0006 | 1.43 | 1.27 | 2,829 | 220 | 4,191,475 | 85.05 | 134.13 | 399 |
| 23 | .0008 | 1.46 | 1.44 | 6,833 | 4,462 | 8,381,775 | 4,148.57 | 209.22 | 699 |

TABLE 11. The c.a. is $G = x_{-1} + x_0x_1 + x_2$, which is bipermutative but not linear.

| k | Fraction Periodic | ν_k | $L^{1/k}$ | P | L | Not-P | Average Period | Average Preperiod | Maximum Preperiod |
|-----|----------------------|---------|-----------|-------|-------|---------|-------------------|----------------------|----------------------|
| 1 | 0.500000 | 1.00 | 1.00 | 1 | 1 | 1 | 1.00 | 0.50 | 1.00 |
| 2 | 0.750000 | 1.73 | 1.00 | 3 | 1 | 1 | 1.00 | 0.25 | 1.00 |
| 3 | 0.500000 | 1.58 | 1.00 | 4 | 1 | 4 | 1.00 | 0.88 | 2.00 |
| 4 | 0.687500 | 1.82 | 1.41 | 11 | 4 | 5 | 2.50 | 0.31 | 1.00 |
| 5 | 0.812500 | 1.91 | 1.71 | 26 | 15 | 6 | 9.75 | 0.19 | 1.00 |
| 6 | 0.281250 | 1.61 | 1.00 | 18 | 1 | 46 | 1.00 | 3.95 | 8.00 |
| 7 | 0.609375 | 1.86 | 1.74 | 78 | 49 | 50 | 37.75 | 0.94 | 5.00 |
| 8 | 0.667969 | 1.90 | 1.81 | 171 | 120 | 85 | 75.94 | 0.86 | 6.00 |
| 9 | 0.482422 | 1.84 | 1.55 | 247 | 54 | 265 | 44.93 | 2.83 | 12.00 |
| 10 | 0.535156 | 1.87 | 1.82 | 548 | 410 | 476 | 345.04 | 2.85 | 17.00 |
| 11 | 0.183105 | 1.71 | 1.60 | 375 | 176 | 1673 | 158.91 | 28.00 | 73.00 |
| 12 | 0.176270 | 1.73 | 1.40 | 722 | 60 | 3374 | 6.38 | 37.95 | 85.00 |
| 13 | 0.200073 | 1.76 | 1.59 | 1639 | 416 | 6553 | 220.46 | 19.73 | 76.00 |
| 14 | 0.212524 | 1.79 | 1.62 | 3482 | 882 | 12902 | 483.97 | 42.97 | 153.00 |
| 15 | 0.231598 | 1.81 | 1.59 | 7589 | 1095 | 25179 | 523.90 | 42.69 | 191.00 |
| 16 | 0.117599 | 1.74 | 1.63 | 7707 | 2688 | 57829 | 1422.26 | 159.56 | 457.00 |
| 17 | 0.078995 | 1.72 | 1.60 | 10354 | 3230 | 120718 | 2481.50 | 371.77 | 938.00 |
| 18 | 0.078449 | 1.73 | 1.37 | 20565 | 324 | 241579 | 302.81 | 350.15 | 1155.00 |
| 19 | 0.061646 | 1.72 | 1.64 | 32320 | 13471 | 491968 | 12128.71 | 404.87 | 1233.00 |
| 20 | 0.065800 | 1.74 | 1.64 | 68996 | 21240 | 979580 | 15870.41 | 285.87 | 1063.00 |
| 21 | 0.032823 | 1.69 | 1.56 | 68835 | 11865 | 2028317 | 816.87 | 1050.92 | 3506.00 |
| 22 | 0.021364 | 1.67 | 1.60 | 89609 | 32428 | 4104695 | 20280.02 | 1335.34 | 5030.00 |
| 23 | 0.011244 | 1.64 | 1.48 | 94324 | 9108 | 8294284 | 7929.18 | 4869.70 | 10024.00 |

TABLE 12. The c.a. is $B = x_0 + x_1x_2$ on the 2-shift. B is degree 1, left permutative and not right closing.

| k | Fraction Periodic | ν_k | $L^{1/k}$ | P | L | Not-P | Average Period | Average Preperiod | Maximum Preperiod |
|-----|----------------------|---------|-----------|---------|--------|-----------|-------------------|----------------------|----------------------|
| 1 | .5000 | 1.00 | 1.00 | 1 | 1 | 1 | 1.00 | 0.50 | 1 |
| 2 | .7500 | 1.73 | 1.00 | 3 | 1 | 1 | 1.00 | 0.25 | 1 |
| 3 | .5000 | 1.58 | 1.00 | 4 | 1 | 4 | 1.00 | 0.88 | 2 |
| 4 | .6875 | 1.82 | 1.00 | 11 | 1 | 5 | 1.00 | 0.31 | 1 |
| 5 | .8125 | 1.91 | 1.37 | 26 | 5 | 6 | 2.56 | 0.19 | 1 |
| 6 | .6562 | 1.86 | 1.61 | 42 | 18 | 22 | 7.38 | 0.86 | 4 |
| 7 | .6093 | 1.86 | 1.60 | 78 | 28 | 50 | 17.35 | 1.16 | 5 |
| 8 | .5117 | 1.83 | 1.62 | 131 | 48 | 125 | 27.50 | 2.08 | 10 |
| 9 | .4296 | 1.82 | 1.58 | 220 | 63 | 292 | 43.61 | 3.39 | 11 |
| 10 | .4082 | 1.82 | 1.31 | 418 | 15 | 606 | 5.45 | 6.91 | 21 |
| 11 | .4355 | 1.85 | 1.57 | 892 | 143 | 1,156 | 99.90 | 12.91 | 53 |
| 12 | .3608 | 1.83 | 1.36 | 1,478 | 42 | 2,618 | 11.61 | 15.59 | 53 |
| 13 | .3270 | 1.83 | 1.66 | 2,679 | 754 | 5,513 | 577.86 | 33.42 | 123 |
| 14 | .2167 | 1.79 | 1.26 | 3,552 | 28 | 12,832 | 23.06 | 79.16 | 191 |
| 15 | .2503 | 1.82 | 1.48 | 8,204 | 385 | 24,564 | 303.28 | 69.75 | 232 |
| 16 | .3152 | 1.86 | 1.63 | 20,659 | 2,528 | 44,877 | 1,197.54 | 48.40 | 281 |
| 17 | .1784 | 1.80 | 1.55 | 23,393 | 1,853 | 107,679 | 1,538.93 | 168.75 | 464 |
| 18 | .1821 | 1.81 | 1.59 | 47,760 | 4,464 | 214,384 | 3,208.77 | 172.00 | 697 |
| 19 | .1357 | 1.80 | 1.49 | 71,175 | 1,957 | 453,113 | 1,685.66 | 352.58 | 1082 |
| 20 | .1620 | 1.82 | 1.56 | 169,886 | 7,976 | 878,690 | 5,604.39 | 258.96 | 953 |
| 21 | .1032 | 1.79 | 1.52 | 216,612 | 7,056 | 1,880,540 | 6,344.22 | 2,389.64 | 4,363 |
| 22 | .0902 | 1.79 | 1.35 | 378,612 | 740 | 3,815,692 | 633.16 | 2,315.42 | 6,465 |
| 23 | .0858 | 1.79 | 1.58 | 720,246 | 39,353 | 7,668,362 | 36,059.28 | 1,760.56 | 5,984 |

TABLE 13. The c.a. is K , the composition which is the automorphism U of Table 9 followed by the left permutative, not right-closing map $x_0 + x_1x_2$. The c.a. K is left closing, not right-closing, and not permutative on either side.

| k | Fraction Periodic | ν_k | $L^{1/k}$ | P | L | Not-P | Average Period | Average Preperiod | Maximum Preperiod |
|-----|-------------------|---------|-----------|--------|-------|---------|----------------|-------------------|-------------------|
| 1 | 0.500 | 1.00 | 1.00 | 1 | 1 | 1 | 1.00 | 0.50 | 1 |
| 2 | 0.750 | 1.73 | 1.00 | 3 | 1 | 1 | 1.00 | 0.25 | 1 |
| 3 | 0.500 | 1.58 | 1.44 | 4 | 3 | 4 | 1.75 | 0.50 | 1 |
| 4 | 0.687 | 1.82 | 1.18 | 11 | 2 | 5 | 1.75 | 0.31 | 1 |
| 5 | 0.812 | 1.91 | 1.71 | 26 | 15 | 6 | 11.00 | 0.19 | 1 |
| 6 | 0.468 | 1.76 | 1.20 | 30 | 3 | 34 | 1.66 | 1.28 | 3 |
| 7 | 0.500 | 1.81 | 1.66 | 64 | 35 | 64 | 28.34 | 0.99 | 4 |
| 8 | 0.667 | 1.90 | 1.63 | 171 | 52 | 85 | 30.39 | 0.52 | 3 |
| 9 | 0.306 | 1.75 | 1.27 | 157 | 9 | 355 | 8.89 | 2.35 | 8 |
| 10 | 0.261 | 1.74 | 1.49 | 268 | 55 | 756 | 20.18 | 6.75 | 18 |
| 11 | 0.387 | 1.83 | 1.57 | 793 | 143 | 1255 | 53.53 | 3.00 | 13 |
| 12 | 0.088 | 1.63 | 1.16 | 362 | 6 | 3734 | 1.39 | 20.61 | 48 |
| 13 | 0.150 | 1.72 | 1.63 | 1236 | 611 | 6956 | 259.75 | 20.15 | 78 |
| 14 | 0.126 | 1.72 | 1.51 | 2068 | 329 | 14316 | 119.61 | 33.22 | 132 |
| 15 | 0.091 | 1.70 | 1.50 | 3014 | 465 | 29754 | 414.94 | 44.45 | 138 |
| 16 | 0.092 | 1.72 | 1.50 | 6043 | 728 | 59493 | 650.33 | 101.66 | 282 |
| 17 | 0.107 | 1.75 | 1.68 | 14145 | 6783 | 116927 | 3918.82 | 48.16 | 196 |
| 18 | 0.060 | 1.71 | 1.58 | 15753 | 4095 | 246391 | 3406.78 | 110.78 | 396 |
| 19 | 0.072 | 1.74 | 1.60 | 38191 | 7619 | 486097 | 6336.19 | 142.98 | 406 |
| 20 | 0.038 | 1.69 | 1.54 | 40396 | 5780 | 1008180 | 1691.96 | 279.69 | 780 |
| 21 | 0.018 | 1.65 | 1.48 | 37867 | 4011 | 2059285 | 3961.81 | 705.45 | 1777 |
| 22 | 0.017 | 1.66 | 1.51 | 75309 | 9658 | 4118995 | 4527.64 | 605.57 | 1770 |
| 23 | 0.017 | 1.67 | 1.57 | 144096 | 34477 | 8244512 | 26857.88 | 1191.56 | 2687 |

TABLE 14. The c.a. is C , the composition $x_0x_1 + x_2$ followed by $x_0 + x_1x_2$, on the 2-shift. The c.a. C is neither left nor right closing. It is not known whether the periodic points of C are dense.

| k | Fraction Periodic | ν_k | $L^{1/k}$ | P | L | Not-P | Average Period | Average Preperiod | Maximum Preperiod |
|-----|----------------------|---------|-----------|---------|--------|--------|-------------------|----------------------|----------------------|
| 1 | 1.00 | 3.00 | 2.00 | 3 | 2 | 0 | 1.67 | 0.00 | 0.00 |
| 2 | 1.00 | 3.00 | 2.44 | 9 | 6 | 0 | 4.56 | 0.00 | 0.00 |
| 3 | 1.00 | 3.00 | 2.46 | 27 | 15 | 0 | 9.30 | 0.00 | 0.00 |
| 4 | 0.11 | 1.73 | 1.56 | 9 | 6 | 72 | 4.56 | 0.89 | 1.00 |
| 5 | 1.00 | 3.00 | 2.77 | 243 | 165 | 0 | 121.13 | 0.00 | 0.00 |
| 6 | 1.00 | 3.00 | 2.80 | 729 | 486 | 0 | 334.54 | 0.00 | 0.00 |
| 7 | 1.00 | 3.00 | 2.57 | 2187 | 742 | 0 | 401.62 | 0.00 | 0.00 |
| 8 | 0.01 | 1.73 | 1.58 | 81 | 40 | 6480 | 33.50 | 4.24 | 9.00 |
| 9 | 1.00 | 3.00 | 2.24 | 19683 | 1469 | 0 | 1185.85 | 0.00 | 0.00 |
| 10 | 1.00 | 3.00 | 2.76 | 59049 | 25865 | 0 | 22737.63 | 0.00 | 0.00 |
| 11 | 1.00 | 3.00 | 2.92 | 177147 | 131857 | 0 | 109208.21 | 0.00 | 0.00 |
| 12 | 0.00 | 1.76 | 1.67 | 909 | 486 | 530532 | 239.27 | 45.51 | 133.00 |
| 13 | 1.00 | 3.00 | 2.79 | 1594323 | 631605 | 0 | 291222.95 | 0.00 | 0.00 |

TABLE 15. This map on the 3-shift is an automorphism W followed by the degree 9 linear map $x_0 + x_2$, where $W = x_0 + 2x_0x_1x_1 + 2x_0x_1 + x_1 * x_1 + x_1$. Let π denote the permutation on $\{0, 1, 2\}$ which transposes 0 and 2. Then $(Wx)_0 = x_0$ if $x_1 \neq 1$ and $(Wx)_0 = \pi(x_0)$ if $x_1 = 1$.

| k | Fraction Periodic | ν_k | $L^{1/k}$ | P | L | Not-P | Average Period | Average Preperiod | Maximum Preperiod |
|-----|----------------------|---------|-----------|--------|------|-----------|-------------------|----------------------|----------------------|
| 1 | .3333 | 1.00 | 1.00 | 1 | 1 | 2 | 1.00 | 1.00 | 2 |
| 2 | .5555 | 2.23 | 1.00 | 5 | 1 | 4 | 1.00 | 0.56 | 2 |
| 3 | .2592 | 1.91 | 1.00 | 7 | 1 | 20 | 1.00 | 1.78 | 3 |
| 4 | .2098 | 2.03 | 1.00 | 17 | 1 | 64 | 1.00 | 3.17 | 8 |
| 5 | .4362 | 2.54 | 2.09 | 106 | 40 | 137 | 27.65 | 1.08 | 4 |
| 6 | .2208 | 2.33 | 1.51 | 161 | 12 | 568 | 6.79 | 2.71 | 9 |
| 7 | .0932 | 2.13 | 1.66 | 204 | 35 | 1,983 | 5.89 | 13.89 | 38 |
| 8 | .0391 | 2.00 | 1.00 | 257 | 1 | 6,304 | 1.00 | 27.02 | 67 |
| 9 | .1667 | 2.45 | 1.60 | 3,283 | 72 | 16,400 | 48.89 | 13.41 | 52 |
| 10 | .0299 | 2.11 | 1.62 | 1,770 | 130 | 57,279 | 62.67 | 55.38 | 163 |
| 11 | .0224 | 2.12 | 1.89 | 3,972 | 1122 | 173,175 | 593.34 | 99.23 | 297 |
| 12 | .0164 | 2.13 | 1.40 | 8,729 | 60 | 522,712 | 12.56 | 88.45 | 222 |
| 13 | .0076 | 2.06 | 1.81 | 12,117 | 2366 | 1,582,206 | 2,228.50 | 676.85 | 1,504 |

TABLE 16. The map $x_0 + x_1x_2$ on the 3-shift: still degree 1, left permutative, not right closing.

| k | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1 | 1.00 | 1.00 | 2.00 | 2.00 | 1.00 | 2.00 | 2.00 | 2.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2 | 0.00 | 0.00 | 0.00 | 1.41 | 1.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | 1.81 | 1.44 | 1.81 | 1.81 | 0.00 | 1.44 | 1.44 | 1.81 | 1.81 | 1.44 | 1.44 | 0.00 | 0.00 |
| 4 | 1.41 | 0.00 | 0.00 | 1.86 | 0.00 | 1.41 | 1.68 | 0.00 | 0.00 | 0.00 | 1.68 | 1.68 | 1.41 |
| 5 | 1.90 | 1.82 | 1.90 | 1.82 | 1.71 | 1.97 | 1.71 | 1.97 | 1.90 | 1.37 | 1.71 | 1.71 | 1.82 |
| 6 | 1.61 | 0.00 | 1.51 | 1.69 | 0.00 | 1.69 | 1.76 | 1.86 | 1.81 | 1.61 | 1.51 | 1.61 | 0.00 |
| 7 | 1.77 | 1.80 | 1.80 | 1.77 | 1.74 | 1.88 | 1.90 | 1.99 | 1.45 | 1.85 | 1.90 | 1.70 | 1.92 |
| 8 | 1.62 | 1.70 | 1.75 | 1.70 | 1.48 | 1.72 | 1.81 | 1.90 | 1.80 | 0.00 | 1.78 | 1.72 | 1.70 |
| 9 | 1.89 | 1.76 | 1.74 | 1.90 | 1.90 | 1.91 | 1.91 | 1.99 | 1.79 | 1.58 | 1.76 | 1.77 | 1.80 |
| 10 | 1.80 | 1.58 | 1.25 | 1.83 | 1.78 | 1.81 | 1.91 | 1.95 | 1.67 | 0.00 | 1.76 | 1.68 | 1.90 |
| 11 | 1.66 | 1.69 | 1.75 | 1.73 | 1.78 | 1.85 | 1.92 | 1.99 | 1.67 | 1.66 | 1.80 | 1.51 | 1.48 |
| 12 | 1.71 | 1.75 | 1.78 | 1.84 | 1.68 | 1.85 | 1.84 | 1.97 | 1.77 | 1.70 | 1.68 | 1.73 | 1.69 |
| 13 | 1.73 | 1.72 | 1.79 | 1.73 | 1.72 | 1.87 | 1.93 | 2.00 | 1.72 | 1.69 | 1.80 | 1.75 | 1.84 |
| 14 | 1.66 | 1.61 | 1.73 | 1.73 | 1.63 | 1.81 | 1.91 | 1.98 | 1.57 | 1.54 | 1.69 | 1.68 | 1.74 |
| 15 | 1.66 | 1.71 | 1.60 | 1.73 | 1.74 | 1.85 | 1.92 | 1.99 | 1.78 | 1.67 | 1.70 | 1.68 | 1.76 |
| 16 | 1.68 | 1.64 | 1.74 | 1.71 | 1.72 | 1.79 | 1.93 | 1.98 | 1.54 | 1.65 | 1.67 | 1.49 | 1.75 |
| 17 | 1.69 | 1.53 | 1.73 | 1.68 | 1.72 | 1.84 | 1.91 | 2.00 | 1.73 | 1.59 | 1.73 | 1.65 | 1.69 |
| 18 | 1.68 | 1.46 | 1.69 | 1.68 | 1.71 | 1.83 | 1.91 | 1.99 | 1.65 | 1.59 | 1.57 | 1.65 | 1.67 |
| 19 | 1.67 | 1.61 | 1.68 | 1.67 | 1.69 | 1.81 | 1.93 | 2.00 | 1.71 | 1.63 | 1.66 | 1.71 | 1.73 |

TABLE 17. $\nu_k^o(\cdot, S_2)$ for the span five maps 1-13 of Table 2.

| k | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1 | 1.00 | 2.00 | 2.00 | 2.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 2.00 | 2.00 | 2.00 |
| 2 | 0.00 | 1.41 | 1.41 | 0.00 | 1.41 | 1.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | 1.81 | 1.44 | 0.00 | 1.44 | 1.44 | 1.44 | 0.00 | 0.00 | 0.00 | 0.00 | 1.81 | 1.81 | 1.81 |
| 4 | 0.00 | 1.68 | 1.86 | 0.00 | 1.41 | 1.68 | 0.00 | 1.41 | 0.00 | 1.68 | 0.00 | 0.00 | 0.00 |
| 5 | 1.82 | 1.82 | 1.82 | 1.71 | 1.71 | 1.90 | 1.71 | 1.58 | 1.71 | 1.58 | 1.97 | 1.97 | 1.97 |
| 6 | 1.81 | 1.86 | 1.76 | 1.61 | 1.51 | 1.51 | 0.00 | 1.69 | 1.81 | 1.81 | 1.69 | 1.69 | 1.69 |
| 7 | 1.80 | 1.85 | 1.92 | 1.60 | 1.70 | 1.83 | 1.83 | 1.54 | 1.70 | 1.54 | 1.99 | 1.99 | 1.99 |
| 8 | 1.41 | 1.83 | 1.70 | 1.72 | 1.83 | 1.86 | 1.68 | 1.68 | 1.54 | 1.68 | 1.81 | 1.81 | 1.86 |
| 9 | 1.84 | 1.84 | 1.76 | 1.76 | 1.71 | 1.78 | 1.78 | 1.74 | 1.84 | 1.87 | 1.99 | 1.99 | 1.99 |
| 10 | 1.86 | 1.85 | 1.83 | 1.65 | 1.76 | 1.82 | 1.79 | 1.52 | 1.67 | 1.72 | 1.90 | 1.90 | 1.93 |
| 11 | 1.57 | 1.78 | 1.81 | 1.88 | 1.81 | 1.75 | 1.77 | 1.64 | 1.79 | 1.54 | 1.99 | 1.99 | 1.99 |
| 12 | 1.57 | 1.83 | 1.84 | 1.66 | 1.64 | 1.80 | 1.62 | 1.73 | 1.73 | 1.67 | 1.94 | 1.94 | 1.94 |
| 13 | 1.71 | 1.71 | 1.68 | 1.73 | 1.72 | 1.85 | 1.61 | 1.78 | 1.79 | 1.65 | 2.00 | 2.00 | 2.00 |
| 14 | 1.76 | 1.73 | 1.72 | 1.67 | 1.66 | 1.80 | 1.57 | 1.73 | 1.75 | 1.60 | 1.95 | 1.95 | 1.96 |
| 15 | 1.71 | 1.76 | 1.79 | 1.72 | 1.77 | 1.78 | 1.72 | 1.65 | 1.66 | 1.52 | 1.99 | 1.99 | 1.99 |
| 16 | 1.71 | 1.71 | 1.71 | 1.73 | 1.69 | 1.78 | 1.71 | 1.51 | 1.65 | 1.55 | 1.97 | 1.97 | 1.97 |
| 17 | 1.74 | 1.66 | 1.63 | 1.64 | 1.71 | 1.78 | 1.64 | 1.65 | 1.67 | 1.68 | 2.00 | 2.00 | 2.00 |
| 18 | 1.71 | 1.68 | 1.68 | 1.67 | 1.62 | 1.73 | 1.65 | 1.61 | 1.52 | 1.62 | 1.98 | 1.98 | 1.98 |
| 19 | 1.68 | 1.69 | 1.70 | 1.70 | 1.63 | 1.75 | 1.70 | 1.66 | 1.65 | 1.60 | 2.00 | 2.00 | 2.00 |

TABLE 18. $\nu_k^o(\cdot, S_2)$ for the span five maps 14-26 of Table 2.

| k | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1 | 2.00 | 2.00 | 1.00 | 1.00 | 2.00 | 2.00 | 2.00 | 1.00 | 2.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.00 | 1.00 | 2.00 |
| 2 | 0.00 | 1.41 | 0.00 | 1.41 | 1.41 | 1.41 | 1.41 | 1.41 | 0.00 | 0.00 | 0.00 | 1.41 | 0.00 | 1.41 | 1.41 | 1.41 |
| 3 | 1.81 | 1.81 | 1.81 | 1.44 | 0.00 | 1.81 | 1.81 | 1.44 | 1.81 | 1.44 | 1.81 | 0.00 | 1.81 | 1.44 | 1.44 | 1.81 |
| 4 | 1.68 | 1.86 | 1.68 | 1.41 | 1.41 | 1.86 | 1.86 | 1.41 | 1.68 | 1.41 | 1.68 | 1.41 | 1.68 | 0.00 | 1.68 | 1.86 |
| 5 | 1.97 | 1.97 | 1.82 | 1.90 | 1.58 | 1.97 | 1.97 | 1.90 | 1.97 | 1.82 | 1.82 | 1.58 | 1.97 | 1.58 | 1.90 | 1.97 |
| 6 | 1.90 | 1.94 | 1.81 | 1.61 | 1.76 | 1.94 | 1.94 | 1.61 | 1.90 | 1.34 | 1.81 | 1.76 | 1.90 | 1.86 | 1.81 | 1.94 |
| 7 | 1.99 | 1.99 | 1.83 | 1.70 | 1.80 | 1.99 | 1.99 | 1.70 | 1.99 | 1.74 | 1.83 | 1.80 | 1.99 | 1.83 | 1.70 | 1.99 |
| 8 | 1.95 | 1.98 | 1.81 | 1.48 | 1.75 | 1.98 | 1.98 | 1.48 | 1.95 | 1.78 | 1.81 | 1.75 | 1.95 | 1.54 | 1.88 | 1.98 |
| 9 | 1.99 | 1.99 | 1.86 | 1.68 | 1.82 | 1.99 | 1.99 | 1.68 | 1.99 | 1.82 | 1.86 | 1.82 | 1.99 | 1.73 | 1.86 | 1.99 |
| 10 | 1.98 | 1.99 | 1.76 | 1.70 | 1.82 | 1.99 | 1.99 | 1.70 | 1.98 | 1.75 | 1.76 | 1.82 | 1.98 | 1.56 | 1.65 | 1.99 |
| 11 | 1.99 | 1.99 | 1.70 | 1.65 | 1.68 | 1.99 | 1.99 | 1.65 | 1.99 | 1.89 | 1.70 | 1.68 | 1.99 | 1.60 | 1.90 | 1.99 |
| 12 | 1.99 | 1.99 | 1.51 | 1.65 | 1.61 | 1.99 | 1.99 | 1.65 | 1.99 | 1.65 | 1.51 | 1.61 | 1.99 | 1.34 | 1.75 | 1.99 |
| 13 | 2.00 | 2.00 | 1.70 | 1.57 | 1.63 | 2.00 | 2.00 | 1.57 | 2.00 | 1.73 | 1.70 | 1.63 | 2.00 | 1.54 | 1.68 | 2.00 |
| 14 | 1.99 | 1.99 | 1.74 | 1.65 | 1.70 | 1.99 | 1.99 | 1.65 | 1.99 | 1.81 | 1.74 | 1.70 | 1.99 | 1.66 | 1.74 | 1.99 |
| 15 | 1.99 | 1.99 | 1.71 | 1.68 | 1.70 | 1.99 | 1.99 | 1.68 | 1.99 | 1.73 | 1.71 | 1.70 | 1.99 | 1.47 | 1.77 | 1.99 |
| 16 | 1.99 | 1.99 | 1.74 | 1.67 | 1.70 | 1.99 | 1.99 | 1.67 | 1.99 | 1.76 | 1.74 | 1.70 | 1.99 | 1.59 | 1.67 | 1.99 |
| 17 | 2.00 | 2.00 | 1.67 | 1.53 | 1.71 | 2.00 | 2.00 | 1.53 | 2.00 | 1.75 | 1.67 | 1.71 | 2.00 | 1.59 | 1.61 | 2.00 |
| 18 | 1.99 | 1.99 | 1.71 | 1.56 | 1.65 | 1.99 | 1.99 | 1.56 | 1.99 | 1.71 | 1.71 | 1.65 | 1.99 | 1.52 | 1.63 | 1.99 |
| 19 | 2.00 | 2.00 | 1.73 | 1.54 | 1.72 | 2.00 | 2.00 | 1.54 | 2.00 | 1.77 | 1.73 | 1.72 | 2.00 | 1.57 | 1.69 | 2.00 |

TABLE 19. $\nu_k^o(\cdot, S_2)$ for the span four maps 1-16 of Table 1.

| k | F1 | F2 | F3 | F4 | F5 | F6 | F7 | F8 | F9 | F10 | F11 | F12 | F13 | F14 | F15 | F16 |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1 | 2.00 | 2.00 | 1.00 | 1.00 | 2.00 | 2.00 | 2.00 | 1.00 | 2.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.00 | 1.00 | 2.00 |
| 2 | 0.00 | 1.41 | 0.00 | 1.41 | 1.41 | 1.41 | 1.41 | 1.41 | 0.00 | 0.00 | 0.00 | 1.41 | 0.00 | 1.41 | 1.41 | 1.41 |
| 3 | 1.81 | 1.81 | 1.81 | 1.44 | 1.44 | 1.81 | 1.81 | 1.44 | 1.81 | 0.00 | 1.81 | 1.44 | 1.81 | 1.44 | 0.00 | 1.81 |
| 4 | 1.41 | 1.86 | 1.68 | 0.00 | 1.41 | 1.86 | 1.86 | 0.00 | 1.41 | 0.00 | 1.68 | 1.41 | 1.41 | 1.41 | 1.68 | 1.86 |
| 5 | 1.97 | 1.97 | 1.82 | 1.58 | 1.71 | 1.97 | 1.97 | 1.58 | 1.97 | 1.71 | 1.82 | 1.71 | 1.97 | 1.90 | 1.90 | 1.97 |
| 6 | 1.86 | 1.94 | 1.69 | 1.86 | 1.69 | 1.94 | 1.94 | 1.86 | 1.86 | 1.69 | 1.69 | 1.69 | 1.86 | 1.61 | 0.00 | 1.94 |
| 7 | 1.99 | 1.99 | 1.66 | 1.83 | 1.70 | 1.99 | 1.99 | 1.83 | 1.99 | 1.60 | 1.66 | 1.70 | 1.99 | 1.70 | 1.92 | 1.99 |
| 8 | 1.93 | 1.98 | 1.81 | 1.54 | 1.80 | 1.98 | 1.98 | 1.54 | 1.93 | 1.41 | 1.81 | 1.80 | 1.93 | 1.48 | 1.75 | 1.98 |
| 9 | 1.99 | 1.99 | 1.71 | 1.73 | 1.62 | 1.99 | 1.99 | 1.73 | 1.99 | 1.77 | 1.71 | 1.62 | 1.99 | 1.68 | 1.68 | 1.99 |
| 10 | 1.95 | 1.99 | 1.70 | 1.56 | 1.44 | 1.99 | 1.99 | 1.56 | 1.95 | 1.79 | 1.70 | 1.44 | 1.95 | 1.70 | 0.00 | 1.99 |
| 11 | 1.99 | 1.99 | 1.74 | 1.60 | 1.46 | 1.99 | 1.99 | 1.60 | 1.99 | 1.59 | 1.74 | 1.46 | 1.99 | 1.65 | 1.65 | 1.99 |
| 12 | 1.97 | 1.99 | 1.65 | 1.34 | 1.55 | 1.99 | 1.99 | 1.34 | 1.97 | 1.63 | 1.65 | 1.55 | 1.97 | 1.65 | 1.69 | 1.99 |
| 13 | 2.00 | 2.00 | 1.75 | 1.54 | 1.65 | 2.00 | 2.00 | 1.54 | 2.00 | 1.53 | 1.75 | 1.65 | 2.00 | 1.57 | 1.67 | 2.00 |
| 14 | 1.98 | 1.99 | 1.74 | 1.66 | 1.53 | 1.99 | 1.99 | 1.66 | 1.98 | 1.72 | 1.74 | 1.53 | 1.98 | 1.65 | 1.51 | 1.99 |
| 15 | 1.99 | 1.99 | 1.74 | 1.47 | 1.64 | 1.99 | 1.99 | 1.47 | 1.99 | 1.68 | 1.74 | 1.64 | 1.99 | 1.68 | 1.74 | 1.99 |
| 16 | 1.99 | 1.99 | 1.66 | 1.59 | 1.55 | 1.99 | 1.99 | 1.59 | 1.99 | 1.66 | 1.66 | 1.55 | 1.99 | 1.67 | 1.68 | 1.99 |
| 17 | 2.00 | 2.00 | 1.67 | 1.59 | 1.57 | 2.00 | 2.00 | 1.59 | 2.00 | 1.74 | 1.67 | 1.57 | 2.00 | 1.53 | 1.69 | 2.00 |
| 18 | 1.99 | 1.99 | 1.63 | 1.52 | 1.61 | 1.99 | 1.99 | 1.52 | 1.99 | 1.70 | 1.63 | 1.61 | 1.99 | 1.56 | 1.61 | 1.99 |
| 19 | 2.00 | 2.00 | 1.69 | 1.57 | 1.51 | 2.00 | 2.00 | 1.57 | 2.00 | 1.54 | 1.69 | 1.51 | 2.00 | 1.54 | 1.63 | 2.00 |

TABLE 20. $\nu_k^o(\cdot, S_2)$ for the span 4 maps 1-16 of Table 1, postcomposed with the flip map $F = x_0 + 1$.

| k | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1 | 1.00 | 1.00 | 2.00 | 2.00 | 1.00 | 2.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.00 | 1.00 |
| 2 | 0.00 | 1.41 | 0.00 | 1.41 | 1.41 | 0.00 | 0.00 | 1.41 | 1.41 | 0.00 | 0.00 | 1.41 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | 0.00 | 0.00 | 0.00 | 0.00 | 1.44 | 0.00 | 1.81 | 0.00 | 0.00 | 1.81 | 1.44 | 1.44 | 0.00 | 0.00 | 0.00 | 1.81 |
| 4 | 0.00 | 1.68 | 1.68 | 1.41 | 1.68 | 1.68 | 1.68 | 1.68 | 1.41 | 1.68 | 1.41 | 0.00 | 1.68 | 1.68 | 0.00 | 1.68 |
| 5 | 1.71 | 1.90 | 1.71 | 1.58 | 1.90 | 1.71 | 1.82 | 1.90 | 1.58 | 1.97 | 1.82 | 1.58 | 1.71 | 1.71 | 1.71 | 1.82 |
| 6 | 1.69 | 0.00 | 1.34 | 1.76 | 1.81 | 1.34 | 1.69 | 0.00 | 1.76 | 1.90 | 1.34 | 1.86 | 1.34 | 1.34 | 1.69 | 1.69 |
| 7 | 1.60 | 1.92 | 1.54 | 1.80 | 1.70 | 1.54 | 1.66 | 1.92 | 1.80 | 1.99 | 1.74 | 1.83 | 1.54 | 1.54 | 1.60 | 1.66 |
| 8 | 1.41 | 1.75 | 1.41 | 1.75 | 1.88 | 1.41 | 1.81 | 1.75 | 1.75 | 1.95 | 1.78 | 1.54 | 1.41 | 1.41 | 1.41 | 1.81 |
| 9 | 1.77 | 1.68 | 1.74 | 1.82 | 1.86 | 1.74 | 1.71 | 1.68 | 1.82 | 1.99 | 1.82 | 1.73 | 1.74 | 1.74 | 1.77 | 1.71 |
| 10 | 1.79 | 0.00 | 1.72 | 1.82 | 1.65 | 1.72 | 1.70 | 0.00 | 1.82 | 1.98 | 1.75 | 1.56 | 1.72 | 1.72 | 1.79 | 1.70 |
| 11 | 1.59 | 1.65 | 1.41 | 1.68 | 1.90 | 1.41 | 1.74 | 1.65 | 1.68 | 1.99 | 1.89 | 1.60 | 1.41 | 1.41 | 1.59 | 1.74 |
| 12 | 1.63 | 1.69 | 1.59 | 1.61 | 1.75 | 1.59 | 1.65 | 1.69 | 1.61 | 1.99 | 1.65 | 1.34 | 1.59 | 1.59 | 1.63 | 1.65 |
| 13 | 1.53 | 1.67 | 1.66 | 1.63 | 1.68 | 1.66 | 1.75 | 1.67 | 1.63 | 2.00 | 1.73 | 1.54 | 1.66 | 1.66 | 1.53 | 1.75 |
| 14 | 1.72 | 1.51 | 1.44 | 1.70 | 1.74 | 1.44 | 1.74 | 1.51 | 1.70 | 1.99 | 1.81 | 1.66 | 1.44 | 1.44 | 1.72 | 1.74 |
| 15 | 1.68 | 1.74 | 1.58 | 1.70 | 1.77 | 1.58 | 1.74 | 1.74 | 1.70 | 1.99 | 1.73 | 1.47 | 1.58 | 1.58 | 1.68 | 1.74 |
| 16 | 1.66 | 1.68 | 1.64 | 1.70 | 1.67 | 1.64 | 1.66 | 1.68 | 1.70 | 1.99 | 1.76 | 1.59 | 1.64 | 1.64 | 1.66 | 1.66 |
| 17 | 1.74 | 1.69 | 1.59 | 1.71 | 1.61 | 1.59 | 1.67 | 1.69 | 1.71 | 2.00 | 1.75 | 1.59 | 1.59 | 1.59 | 1.74 | 1.67 |
| 18 | 1.70 | 1.61 | 1.46 | 1.65 | 1.63 | 1.46 | 1.63 | 1.61 | 1.65 | 1.99 | 1.71 | 1.52 | 1.46 | 1.46 | 1.70 | 1.63 |
| 19 | 1.54 | 1.63 | 1.60 | 1.72 | 1.69 | 1.60 | 1.69 | 1.63 | 1.72 | 2.00 | 1.77 | 1.57 | 1.60 | 1.60 | 1.54 | 1.69 |

TABLE 21. $\nu_k^o(\cdot, S_2)$ for the span 4 onto maps 17-32 of Table 1.

| k | $F17$ | $F18$ | $F19$ | $F20$ | $F21$ | $F22$ | $F23$ | $F24$ | $F25$ | $F26$ | $F27$ | $F28$ | $F29$ | $F30$ | $F31$ | $F32$ |
|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1 | 1.00 | 1.00 | 2.00 | 2.00 | 1.00 | 2.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.00 | 1.00 |
| 2 | 0.00 | 1.41 | 0.00 | 1.41 | 1.41 | 0.00 | 0.00 | 1.41 | 1.41 | 0.00 | 0.00 | 1.41 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | 1.44 | 1.44 | 1.44 | 1.44 | 0.00 | 1.44 | 1.81 | 1.44 | 1.44 | 1.81 | 0.00 | 1.44 | 1.44 | 1.44 | 1.44 | 1.81 |
| 4 | 1.41 | 1.68 | 1.68 | 1.41 | 1.68 | 1.68 | 1.68 | 1.68 | 1.41 | 1.41 | 0.00 | 1.41 | 1.68 | 1.68 | 1.41 | 1.68 |
| 5 | 1.82 | 1.90 | 1.58 | 1.71 | 1.90 | 1.58 | 1.82 | 1.90 | 1.71 | 1.97 | 1.71 | 1.90 | 1.58 | 1.58 | 1.82 | 1.82 |
| 6 | 1.34 | 1.81 | 1.61 | 1.69 | 0.00 | 1.61 | 1.81 | 1.81 | 1.69 | 1.86 | 1.69 | 1.61 | 1.61 | 1.61 | 1.34 | 1.81 |
| 7 | 1.74 | 1.70 | 1.70 | 1.70 | 1.92 | 1.70 | 1.83 | 1.70 | 1.70 | 1.99 | 1.60 | 1.70 | 1.70 | 1.70 | 1.74 | 1.83 |
| 8 | 1.78 | 1.88 | 1.65 | 1.80 | 1.75 | 1.65 | 1.81 | 1.88 | 1.80 | 1.93 | 1.41 | 1.48 | 1.65 | 1.65 | 1.78 | 1.81 |
| 9 | 1.82 | 1.86 | 1.60 | 1.62 | 1.68 | 1.60 | 1.86 | 1.86 | 1.62 | 1.99 | 1.77 | 1.68 | 1.60 | 1.60 | 1.82 | 1.86 |
| 10 | 1.75 | 1.65 | 1.61 | 1.44 | 0.00 | 1.61 | 1.76 | 1.65 | 1.44 | 1.95 | 1.79 | 1.70 | 1.61 | 1.61 | 1.75 | 1.76 |
| 11 | 1.89 | 1.90 | 1.69 | 1.46 | 1.65 | 1.69 | 1.70 | 1.90 | 1.46 | 1.99 | 1.59 | 1.65 | 1.69 | 1.69 | 1.89 | 1.70 |
| 12 | 1.65 | 1.75 | 1.49 | 1.55 | 1.69 | 1.49 | 1.51 | 1.75 | 1.55 | 1.97 | 1.63 | 1.65 | 1.49 | 1.49 | 1.65 | 1.51 |
| 13 | 1.73 | 1.68 | 1.66 | 1.65 | 1.67 | 1.66 | 1.70 | 1.68 | 1.65 | 2.00 | 1.53 | 1.57 | 1.66 | 1.66 | 1.73 | 1.70 |
| 14 | 1.81 | 1.74 | 1.59 | 1.53 | 1.51 | 1.59 | 1.74 | 1.74 | 1.53 | 1.98 | 1.72 | 1.65 | 1.59 | 1.59 | 1.81 | 1.74 |
| 15 | 1.73 | 1.77 | 1.53 | 1.64 | 1.74 | 1.53 | 1.71 | 1.77 | 1.64 | 1.99 | 1.68 | 1.68 | 1.53 | 1.53 | 1.73 | 1.71 |
| 16 | 1.76 | 1.67 | 1.69 | 1.55 | 1.68 | 1.69 | 1.74 | 1.67 | 1.55 | 1.99 | 1.66 | 1.67 | 1.69 | 1.69 | 1.76 | 1.74 |
| 17 | 1.75 | 1.61 | 1.63 | 1.57 | 1.69 | 1.63 | 1.67 | 1.61 | 1.57 | 2.00 | 1.74 | 1.53 | 1.63 | 1.63 | 1.75 | 1.67 |
| 18 | 1.71 | 1.63 | 1.55 | 1.61 | 1.61 | 1.55 | 1.71 | 1.63 | 1.61 | 1.99 | 1.70 | 1.56 | 1.55 | 1.55 | 1.71 | 1.71 |
| 19 | 1.77 | 1.69 | 1.65 | 1.51 | 1.63 | 1.65 | 1.73 | 1.69 | 1.51 | 2.00 | 1.54 | 1.54 | 1.65 | 1.65 | 1.77 | 1.73 |

TABLE 22. $\nu_k^o(\cdot, S_2)$ for the span 4 maps 17-32 of Table 1, post-composed with the flip map $F = x_0 + 1$.

| k | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1 | 1.00 | 1.00 | 2.00 | 2.00 | 1.00 | 1.00 | 1.00 | 2.00 | 1.00 | 2.00 | 2.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.00 |
| 2 | 1.41 | 0.00 | 1.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.41 | 1.41 | 1.41 | 0.00 | 1.41 | 0.00 | 0.00 | 0.00 |
| 3 | 1.44 | 0.00 | 1.44 | 0.00 | 0.00 | 1.44 | 1.44 | 1.81 | 0.00 | 0.00 | 1.44 | 1.81 | 0.00 | 1.81 | 1.44 | 1.44 |
| 4 | 0.00 | 1.41 | 1.41 | 1.68 | 1.68 | 0.00 | 1.86 | 1.41 | 0.00 | 0.00 | 1.41 | 1.41 | 1.41 | 1.41 | 1.68 | 1.86 |
| 5 | 1.37 | 1.37 | 1.58 | 1.71 | 1.82 | 1.90 | 1.37 | 1.71 | 1.90 | 1.71 | 1.58 | 1.82 | 1.37 | 1.71 | 1.37 | 1.90 |
| 6 | 1.51 | 1.51 | 1.51 | 1.34 | 1.51 | 1.34 | 1.34 | 0.00 | 0.00 | 1.51 | 0.00 | 1.61 | 1.34 | 0.00 | 1.34 | 0.00 |
| 7 | 1.74 | 1.88 | 1.54 | 1.60 | 1.66 | 1.60 | 1.77 | 1.54 | 1.70 | 1.66 | 0.00 | 1.74 | 1.80 | 1.54 | 1.60 | 1.32 |
| 8 | 1.48 | 1.58 | 1.65 | 1.62 | 1.41 | 1.65 | 0.00 | 1.62 | 1.62 | 1.58 | 1.68 | 1.54 | 0.00 | 1.62 | 1.68 | 0.00 |
| 9 | 1.62 | 1.78 | 1.74 | 1.58 | 1.74 | 1.58 | 1.68 | 0.00 | 1.48 | 1.48 | 1.44 | 1.73 | 1.44 | 0.00 | 1.64 | 1.60 |
| 10 | 1.34 | 1.62 | 1.40 | 1.54 | 1.66 | 1.58 | 1.52 | 1.68 | 1.61 | 1.62 | 1.34 | 1.47 | 1.34 | 1.68 | 1.34 | 1.50 |
| 11 | 1.58 | 1.69 | 1.43 | 1.64 | 1.51 | 1.76 | 1.67 | 1.60 | 1.55 | 1.53 | 1.60 | 1.66 | 1.58 | 1.60 | 1.57 | 1.67 |
| 12 | 1.54 | 1.44 | 1.60 | 1.51 | 1.63 | 1.74 | 1.46 | 1.56 | 0.00 | 1.54 | 1.54 | 1.44 | 1.52 | 1.56 | 1.38 | 1.38 |
| 13 | 1.61 | 1.66 | 1.62 | 1.65 | 1.56 | 1.77 | 1.59 | 1.63 | 1.32 | 1.66 | 1.61 | 1.47 | 1.57 | 1.63 | 1.68 | 1.60 |
| 14 | 1.64 | 1.62 | 1.52 | 1.57 | 1.53 | 1.80 | 1.55 | 1.40 | 1.55 | 1.41 | 1.60 | 1.55 | 1.48 | 1.40 | 1.55 | 1.65 |
| 15 | 1.60 | 1.74 | 1.60 | 1.38 | 1.62 | 1.73 | 1.58 | 1.62 | 1.52 | 1.52 | 1.50 | 1.49 | 1.66 | 1.62 | 1.66 | 1.61 |
| 16 | 1.48 | 1.73 | 1.47 | 1.53 | 1.50 | 1.60 | 1.51 | 1.59 | 1.49 | 1.44 | 1.29 | 1.58 | 1.57 | 1.59 | 1.49 | 1.57 |
| 17 | 1.46 | 1.69 | 1.58 | 1.59 | 1.62 | 1.64 | 1.61 | 1.47 | 1.60 | 1.58 | 1.56 | 1.55 | 1.46 | 1.47 | 1.63 | 1.36 |
| 18 | 1.56 | 1.72 | 1.55 | 1.49 | 1.55 | 1.52 | 1.46 | 1.50 | 1.51 | 1.52 | 1.54 | 1.45 | 1.56 | 1.50 | 1.52 | 1.45 |
| 19 | 1.56 | 1.66 | 1.56 | 1.55 | 1.52 | 1.64 | 1.45 | 1.60 | 1.47 | 1.51 | 1.55 | 1.56 | 1.52 | 1.60 | 1.59 | 1.48 |

TABLE 23. ν_k^c for 16 left permutative span 5 maps.

Let $p_n(x[0, 4])$ denote the polynomial rule for the map n in Table 23 and let $q_n(x[0, 3])$ denote the polynomial rule for the span 4 map n in Table 1. Then p_n is defined by $p_n(x[0, 4]) = x_0 + q_n(x[1, 4])$. The purpose of Table 23 is to allow a rough comparison of a sample of maps which are linear in an end variable to maps which are not (Table 23 vs. Table 1). We see no particular difference.

Table 24 gives complete cycle data for the c.a. B through shift period $k = 22$. For each k , all B -periods p of points from $P_k(S_2)$ are listed. The multiplicities given are the number μ_{orb} of B -cycles in $P_k(S_2)$ with the given size p ; the number μ_{per} of points in all these cycles; and the number μ_{ev} of points in $P_k(S_2)$ with eventual period p .

| k | p | μ_{orb} | μ_{per} | μ_{ev} | k | p | μ_{orb} | μ_{per} | μ_{ev} |
|----|------|-------------|-------------|------------|----|-------|-------------|-------------|------------|
| 1 | 1 | 1 | 1 | 2 | 16 | 1 | 2207 | 2207 | 2208 |
| 2 | 1 | 3 | 3 | 4 | | 4 | 1 | 4 | 6192 |
| 3 | 1 | 4 | 4 | 8 | | 120 | 1 | 120 | 23520 |
| 4 | 1 | 7 | 7 | 8 | | 2688 | 2 | 5376 | 33616 |
| | 4 | 1 | 4 | 8 | 17 | 1 | 3571 | 3571 | 3572 |
| 5 | 1 | 11 | 11 | 12 | | 1020 | 1 | 1020 | 1530 |
| | 15 | 1 | 15 | 20 | | 2533 | 1 | 2533 | 119357 |
| 6 | 1 | 18 | 18 | 64 | | 3230 | 1 | 3230 | 6613 |
| 7 | 1 | 29 | 29 | 30 | 18 | 1 | 5778 | 5778 | 5824 |
| | 49 | 1 | 49 | 98 | | 9 | 1 | 9 | 9 |
| 8 | 1 | 47 | 47 | 48 | | 38 | 36 | 1368 | 3834 |
| | 4 | 1 | 4 | 48 | | 54 | 3 | 162 | 4815 |
| | 120 | 1 | 120 | 160 | | 108 | 6 | 648 | 648 |
| 9 | 1 | 76 | 76 | 80 | | 216 | 6 | 1296 | 7740 |
| | 9 | 1 | 9 | 9 | | 296 | 36 | 10656 | 10656 |
| | 54 | 3 | 162 | 423 | | 324 | 2 | 648 | 228618 |
| 10 | 1 | 123 | 123 | 124 | 19 | 1 | 9349 | 9349 | 9350 |
| | 15 | 1 | 15 | 40 | | 76 | 1 | 76 | 76 |
| | 410 | 1 | 410 | 860 | | 133 | 2 | 266 | 14421 |
| 11 | 1 | 199 | 199 | 200 | | 171 | 1 | 171 | 171 |
| | 176 | 1 | 176 | 1848 | | 646 | 1 | 646 | 2755 |
| 12 | 1 | 322 | 322 | 3692 | | 4161 | 1 | 4161 | 25156 |
| | 4 | 1 | 4 | 8 | | 4180 | 1 | 4180 | 12122 |
| | 56 | 6 | 336 | 336 | | 13471 | 1 | 13471 | 460237 |
| | 60 | 1 | 60 | 60 | 20 | 1 | 15127 | 15127 | 15128 |
| 13 | 1 | 521 | 521 | 522 | | 4 | 1 | 4 | 8 |
| | 10 | 13 | 130 | 650 | | 15 | 1 | 15 | 1000 |
| | 26 | 1 | 26 | 117 | | 132 | 5 | 660 | 1560 |
| | 143 | 1 | 143 | 3900 | | 140 | 4 | 560 | 560 |
| | 403 | 1 | 403 | 845 | | 410 | 1 | 410 | 9420 |
| | 416 | 1 | 416 | 2158 | | 5240 | 2 | 10480 | 306300 |
| 14 | 1 | 843 | 843 | 844 | | 20500 | 1 | 20500 | 197240 |
| | 49 | 1 | 49 | 602 | | 21240 | 1 | 21240 | 517360 |
| | 161 | 2 | 322 | 2212 | 21 | 1 | 24476 | 24476 | 24480 |
| | 448 | 2 | 896 | 7686 | | 14 | 3 | 42 | 42 |
| | 490 | 1 | 490 | 882 | | 21 | 2 | 42 | 42 |
| | 882 | 1 | 882 | 4158 | | 49 | 1 | 49 | 98 |
| 15 | 1 | 1364 | 1364 | 1368 | | 57 | 21 | 1197 | 1197 |
| | 15 | 1 | 15 | 20 | | 266 | 3 | 798 | 1949766 |
| | 180 | 1 | 180 | 180 | | 2618 | 6 | 15708 | 15708 |
| | 399 | 5 | 1995 | 8625 | | 4886 | 3 | 14658 | 14658 |
| | 450 | 3 | 1350 | 15705 | | 11865 | 1 | 11865 | 91161 |
| | 530 | 3 | 1590 | 1590 | 22 | 1 | 39603 | 39603 | 39604 |
| | 1095 | 1 | 1095 | 5280 | | 132 | 2 | 264 | 264 |
| | | | | | | 176 | 1 | 176 | 910272 |
| | | | | | | 660 | 1 | 660 | 1100 |
| | | | | | | 1067 | 2 | 2134 | 112948 |
| | | | | | | 14344 | 1 | 14344 | 924814 |
| | | | | | | 32428 | 1 | 32428 | 2205302 |

TABLE 24. Table for $B = x_0 + x_1x_2$ constructed from FPeriod: complete data for shift-periods through $k = 22$. See the adjacent text for definitions of column items.

4. FPROBPERIOD TABLES

For a given map f , a given positive integer N and a given positive integer m and given set of positive integers k , the program FProbPeriod will for each k randomly sample m blocks of length k on alphabet $\{0, 1, \dots, N-1\}$, and compute the period and preperiod under f of the point x in $P_k(S_N)$ such that $x[0, k-1]$ is the chosen block. Here, by definition the period p of x is the eventual period: the smallest $j > 0$ such that for some $k \geq 0$, $f^k(x) = f^{k+j}(x)$. So, p is the length of the f -cycle into which x is mapped by some f^k , $k \geq 0$. The preperiod of x is the smallest nonnegative k such that $f^k(x)$ is f -periodic, that is $f^k(x) = f^{k+p}(x)$. For a given sample of m points from $P_k(S_2)$, the multiplicity μ of p is the number of times the sampled point has the (eventual) period p . In the following tables, we restrict to cellular automata f on $N = 2$ symbols.

| k | p | μ | k | p | μ | k | p | μ |
|-----|-----|-------|-----|--------|-------|-----|----------|-------|
| 1 | 1 | 10 | 15 | 1 | 1 | 27 | 3402 | 4 |
| 2 | 1 | 10 | | 399 | 3 | | 12096 | 3 |
| 3 | 1 | 10 | | 450 | 3 | | 218835 | 2 |
| 4 | 1 | 4 | | 530 | 1 | | 242352 | 1 |
| | 4 | 6 | | 1095 | 2 | 28 | 882 | 1 |
| 5 | 1 | 4 | 16 | 120 | 7 | | 32144 | 2 |
| | 15 | 6 | | 2688 | 3 | | 57036 | 7 |
| 6 | 1 | 10 | 17 | 2533 | 9 | 29 | 98223 | 2 |
| 7 | 1 | 4 | | 3230 | 1 | | 193256 | 1 |
| | 49 | 6 | 18 | 296 | 1 | | 340286 | 3 |
| 8 | 1 | 2 | | 324 | 9 | | 504252 | 4 |
| | 4 | 2 | 19 | 1 | 1 | 30 | 17580 | 1 |
| | 120 | 6 | | 13471 | 9 | | 161721 | 8 |
| 9 | 1 | 2 | 20 | 5240 | 3 | | 212670 | 1 |
| | 9 | 1 | | 20500 | 1 | 31 | 2228621 | 10 |
| | 54 | 7 | | 21240 | 6 | 32 | 473792 | 10 |
| 10 | 410 | 10 | 21 | 266 | 10 | 33 | 74439 | 7 |
| 11 | 1 | 1 | 22 | 176 | 2 | | 313984 | 3 |
| | 176 | 9 | | 14344 | 4 | 34 | 2533 | 10 |
| 12 | 1 | 8 | | 32428 | 4 | 35 | 1635074 | 6 |
| | 56 | 2 | 23 | 1 | 2 | | 4485250 | 1 |
| 13 | 1 | 1 | | 622 | 1 | | 14840595 | 3 |
| | 143 | 3 | | 9108 | 7 | 36 | 152 | 2 |
| | 403 | 2 | 24 | 1 | 2 | | 324 | 2 |
| | 416 | 4 | | 12432 | 4 | | 22974 | 1 |
| 14 | 49 | 1 | | 20256 | 4 | | 1700772 | 1 |
| | 448 | 7 | 25 | 61830 | 4 | | 4191696 | 4 |
| | 882 | 2 | | 104425 | 6 | 37 | 1365226 | 1 |
| | | | 26 | 143 | 1 | | 7065594 | 5 |
| | | | | 6994 | 9 | | 39209196 | 4 |

TABLE 25. FProbPeriod output for the c.a. $B = x_0 + x_1x_2$ on two symbols, with sample size $m = 10$.

| k | p_{30} | μ_{30} | p_{10} | μ_{10} | k | p_{30} | μ_{30} | p_{10} | μ_{10} | k | p_{30} | μ_{30} | p_{10} | μ_{10} | |
|-----|----------|------------|----------|------------|-------|----------|------------|----------|------------|--------|----------|------------|----------|------------|---|
| 1 | 1 | 30 | 1 | 10 | 15 | 1 | 4 | | | 23 | 1 | 1 | | | |
| 2 | 1 | 30 | 1 | 10 | | 180 | 1 | | | | 622 | 2 | 622 | 1 | |
| 3 | 1 | 30 | 1 | 10 | | 399 | 10 | 399 | 6 | | 9108 | 27 | 9108 | 9 | |
| 4 | 1 | 13 | 1 | 5 | 16 | 450 | 8 | 450 | 1 | 24 | 1 | 4 | | | |
| | 4 | 17 | 4 | 5 | | 530 | 2 | | | | 184 | 1 | | | |
| 5 | 1 | 12 | 1 | 4 | | 1095 | 5 | 1095 | 3 | | 2330 | 1 | 2330 | 1 | |
| | 15 | 18 | 15 | 6 | 1 | 2 | | | 7440 | 6 | 7440 | 1 | | | |
| 6 | 1 | 30 | 1 | 10 | 17 | 4 | 1 | | | 25 | 12432 | 10 | 12432 | 5 | |
| | 7 | 1 | 9 | 1 | | 3 | 120 | 12 | 120 | | 3 | 20256 | 8 | 20256 | 3 |
| 49 | | 21 | 49 | 7 | | 2688 | 15 | 2688 | 7 | | 4325 | 1 | | | |
| 8 | 1 | 2 | 1 | 3 | 18 | 1 | 2 | | | 26 | 13015 | 1 | | | |
| | 4 | 6 | 4 | 2 | | 2533 | 26 | 2533 | 9 | | 61830 | 14 | 61830 | 4 | |
| | 120 | 22 | 120 | 5 | | 3230 | 2 | 3230 | 1 | | 73175 | 4 | 73175 | 1 | |
| 9 | 1 | 3 | | | 19 | 1 | 1 | 54 | 1 | 27 | 104425 | 10 | 104425 | 5 | |
| | 54 | 27 | 54 | 10 | | 216 | 2 | | | | 6994 | 30 | 6994 | 10 | |
| 10 | 1 | 2 | | | | 296 | 5 | | | | 3402 | 9 | 3402 | 3 | |
| | 15 | 3 | 1 | 2 | 324 | 22 | 324 | 9 | 12096 | 9 | 12096 | 3 | | | |
| | 410 | 25 | 410 | 8 | 4161 | 1 | 1 | 1 | 218835 | 10 | 218835 | 1 | | | |
| 11 | 1 | 9 | 1 | 2 | 20 | 4180 | 2 | 4180 | 1 | 28 | 242352 | 2 | 242352 | 3 | |
| | 176 | 21 | 176 | 8 | | 13471 | 27 | 13471 | 8 | | 448 | 5 | | | |
| 12 | 1 | 27 | 1 | 10 | | 15 | 1 | | | | 882 | 5 | 882 | 4 | |
| | 56 | 3 | | | 5240 | 5 | 5240 | 5 | 32144 | 6 | 32144 | 1 | | | |
| | 13 | 1 | 1 | | | 20500 | 7 | 20500 | 1 | 57036 | 14 | 57036 | 5 | | |
| 10 | | 2 | 10 | 1 | 21240 | 17 | 21240 | 4 | 29 | 98223 | 4 | | | | |
| 143 | | 13 | 143 | 2 | 1 | 1 | | | | 193256 | 7 | | | | |
| 403 | 5 | 403 | 1 | 266 | 26 | 266 | 10 | 340286 | | 13 | 340286 | 3 | | | |
| 14 | 416 | 9 | 416 | 6 | 21 | 11865 | 3 | | | 30 | 504252 | 6 | 504252 | 7 | |
| | 49 | 2 | | | | 176 | 6 | 176 | 4 | | 1 | 1 | | | |
| | | 161 | 4 | 161 | | 3 | 1067 | 1 | | | | 450 | 5 | | |
| | 448 | 11 | 448 | 4 | 22 | 14344 | 9 | | | 1800 | 1 | 1800 | 1 | | |
| | 490 | 1 | 490 | 1 | | 32428 | 14 | 32428 | 6 | 127995 | 2 | | | | |
| | 882 | 12 | 882 | 2 | | | | | | 132720 | 1 | | | | |
| | | | | | | | | | 161721 | 19 | 161721 | 7 | | | |
| | | | | | | | | | 212670 | 1 | 212670 | 2 | | | |

TABLE 26. Table for $B = x_0 + x_1x_2$ constructed as for Table 29, for sample sizes 10 and 30 for FProbPeriod. The longest orbit length p found is the same for both sample sizes, except for $k = 12$ and $k = 21$. Exact cycle data for the map B , through shift period $k = 22$, is given in Table 24. Note that for $k \leq 22$, only at $k = 12$ did the size-30 probabilistic sample miss the largest B period.

| map | A | B | C | E | G | H | J | K |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| k | $L^{1/k}$ | $L^{1/k}$ | $L^{1/k}$ | $L^{1/k}$ | $L^{1/k}$ | $L^{1/k}$ | $L^{1/k}$ | $L^{1/k}$ |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 3 | 1.44 | 1.00 | 1.44 | 1.00 | 1.00 | 1.44 | 1.44 | 1.00 |
| 4 | 1.00 | 1.41 | 1.18 | 1.41 | 1.00 | 1.00 | 1.41 | 1.00 |
| 5 | 1.71 | 1.71 | 1.71 | 1.58 | 1.37 | 1.37 | 1.58 | 1.37 |
| 6 | 1.34 | 1.00 | 1.20 | 1.00 | 1.51 | 1.20 | 1.61 | 1.61 |
| 7 | 1.32 | 1.74 | 1.66 | 1.54 | 1.32 | 1.60 | 1.32 | 1.60 |
| 8 | 1.00 | 1.81 | 1.63 | 1.18 | 1.00 | 1.29 | 1.18 | 1.62 |
| 9 | 1.58 | 1.55 | 1.27 | 1.22 | 1.48 | 1.29 | 1.58 | 1.58 |
| 10 | 1.40 | 1.82 | 1.49 | 1.46 | 1.00 | 1.31 | 1.25 | 1.31 |
| 11 | 1.69 | 1.60 | 1.57 | 1.55 | 1.53 | 1.43 | 1.46 | 1.57 |
| 12 | 1.23 | 1.39 | 1.16 | 1.00 | 1.30 | 1.34 | 1.30 | 1.36 |
| 13 | 1.67 | 1.59 | 1.63 | 1.49 | 1.53 | 1.48 | 1.53 | 1.66 |
| 14 | 1.20 | 1.62 | 1.38 | 1.35 | 1.37 | 1.38 | 1.39 | 1.26 |
| 15 | 1.19 | 1.59 | 1.50 | 1.39 | 1.40 | 1.42 | 1.44 | 1.48 |
| 16 | 1.00 | 1.63 | 1.50 | 1.47 | 1.40 | 1.13 | 1.54 | 1.63 |
| 17 | 1.38 | 1.60 | 1.68 | 1.55 | 1.45 | 1.52 | 1.52 | 1.55 |
| 18 | 1.30 | 1.37 | 1.58 | 1.41 | 1.33 | 1.40 | 1.53 | 1.59 |
| 19 | 1.62 | 1.64 | 1.60 | 1.45 | 1.50 | 1.51 | 1.47 | 1.49 |
| 20 | 1.22 | 1.64 | 1.54 | 1.50 | 1.32 | 1.18 | 1.12 | 1.56 |
| 21 | 1.21 | 1.30 | 1.48 | 1.36 | 1.15 | 1.29 | 1.48 | 1.52 |
| 22 | 1.34 | 1.60 | 1.51 | 1.46 | 1.23 | 1.49 | 1.40 | 1.35 |
| 23 | 1.39 | 1.48 | 1.57 | 1.53 | 1.44 | 1.55 | 1.53 | 1.58 |
| 24 | 1.14 | 1.51 | 1.54 | 1.45 | 1.39 | 1.14 | 1.46 | 1.46 |
| 25 | 1.50 | 1.58 | 1.48 | 1.50 | 1.23 | 1.53 | 1.44 | 1.56 |
| 26 | 1.32 | 1.40 | 1.56 | 1.46 | 1.33 | 1.44 | 1.43 | 1.29 |
| 27 | 1.42 | 1.58 | 1.54 | 1.50 | 1.43 | 1.46 | 1.28 | 1.58 |
| 28 | 1.12 | 1.47 | 1.43 | 1.47 | 1.30 | 1.31 | 1.48 | 1.44 |
| 29 | 1.56 | 1.57 | 1.54 | 1.49 | 1.31 | 1.35 | 1.51 | 1.63 |
| 30 | 1.12 | 1.50 | 1.59 | 1.44 | 1.33 | 1.37 | 1.45 | 1.64 |
| 31 | 1.11 | 1.60 | 1.59 | 1.41 | 1.47 | 1.32 | 1.49 | 1.63 |
| 32 | 1.00 | 1.50 | 1.60 | 1.26 | 1.31 | 1.46 | 1.48 | 1.54 |
| 33 | 1.23 | 1.46 | 1.63 | 1.44 | 1.37 | 1.43 | 1.49 | 1.54 |
| 34 | 1.20 | 1.25 | 1.45 | 1.48 | 1.35 | 1.48 | 1.48 | 1.56 |
| 35 | 1.26 | 1.60 | 1.44 | 1.48 | 1.42 | 1.43 | 1.48 | 1.59 |
| 36 | 1.16 | 1.52 | 1.50 | 1.41 | 1.27 | 1.29 | 1.38 | 1.46 |
| 37 | 1.49 | 1.60 | 1.55 | 1.46 | 1.30 | 1.44 | 1.45 | 1.57 |

TABLE 27. This table is based at each k on a random sample of 10 blocks of length k on two symbols. L is the maximum period from the sample.

$A = x_0 + x_1$, linear, and $B = x_0 + x_1x_2$, permutative.

$C = B \circ B_{rev}$, nonclosing, where $B_{rev} = x_0x_1 + x_2$.

$G = x_{-1} + x_0x_1 + x_2$, bipermutative, nonlinear.

$E = B \circ A$, degree 2, left closing, not right closing.

$J = A \circ U$, where $U = x_0 + x_{-2}(1 + x_{-1})x_1x_2$ is invertible.

$H = A \circ A \circ U$, and $K = B \circ U$.

| map | A | B | C | E | G | H | J | K |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| k | $M^{1/k}$ | $M^{1/k}$ | $M^{1/k}$ | $M^{1/k}$ | $M^{1/k}$ | $M^{1/k}$ | $M^{1/k}$ | $M^{1/k}$ |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 |
| 2 | 1.41 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.41 | 1.00 |
| 3 | 1.00 | 1.25 | 1.00 | 1.25 | 1.25 | 1.00 | 1.00 | 1.25 |
| 4 | 1.41 | 1.00 | 1.00 | 1.18 | 1.31 | 1.18 | 1.31 | 1.00 |
| 5 | 1.00 | 1.00 | 1.00 | 1.00 | 1.31 | 1.24 | 1.14 | 1.00 |
| 6 | 1.12 | 1.41 | 1.20 | 1.25 | 1.20 | 1.12 | 1.12 | 1.25 |
| 7 | 1.00 | 1.21 | 1.16 | 1.10 | 1.34 | 1.16 | 1.32 | 1.25 |
| 8 | 1.29 | 1.09 | 1.00 | 1.33 | 1.36 | 1.18 | 1.36 | 1.31 |
| 9 | 1.00 | 1.25 | 1.25 | 1.27 | 1.19 | 1.24 | 1.24 | 1.24 |
| 10 | 1.07 | 1.14 | 1.32 | 1.11 | 1.30 | 1.21 | 1.30 | 1.30 |
| 11 | 1.00 | 1.44 | 1.23 | 1.26 | 1.28 | 1.25 | 1.25 | 1.42 |
| 12 | 1.12 | 1.44 | 1.37 | 1.34 | 1.26 | 1.20 | 1.32 | 1.38 |
| 13 | 1.00 | 1.34 | 1.27 | 1.32 | 1.21 | 1.25 | 1.23 | 1.41 |
| 14 | 1.05 | 1.41 | 1.37 | 1.32 | 1.30 | 1.25 | 1.35 | 1.42 |
| 15 | 1.00 | 1.39 | 1.37 | 1.30 | 1.26 | 1.34 | 1.32 | 1.41 |
| 16 | 1.18 | 1.46 | 1.39 | 1.34 | 1.28 | 1.25 | 1.29 | 1.32 |
| 17 | 1.00 | 1.47 | 1.34 | 1.33 | 1.28 | 1.26 | 1.26 | 1.41 |
| 18 | 1.03 | 1.46 | 1.34 | 1.31 | 1.23 | 1.22 | 1.30 | 1.40 |
| 19 | 1.00 | 1.44 | 1.34 | 1.32 | 1.25 | 1.34 | 1.32 | 1.42 |
| 20 | 1.07 | 1.39 | 1.37 | 1.28 | 1.28 | 1.27 | 1.36 | 1.37 |
| 21 | 1.00 | 1.46 | 1.41 | 1.36 | 1.33 | 1.35 | 1.33 | 1.48 |
| 22 | 1.03 | 1.44 | 1.39 | 1.34 | 1.26 | 1.23 | 1.31 | 1.48 |
| 23 | 1.00 | 1.47 | 1.40 | 1.33 | 1.29 | 1.34 | 1.31 | 1.44 |
| 24 | 1.09 | 1.43 | 1.43 | 1.35 | 1.32 | 1.28 | 1.29 | 1.44 |
| 25 | 1.00 | 1.44 | 1.41 | 1.33 | 1.30 | 1.27 | 1.30 | 1.47 |
| 26 | 1.02 | 1.45 | 1.43 | 1.31 | 1.28 | 1.26 | 1.31 | 1.45 |
| 27 | 1.00 | 1.45 | 1.43 | 1.33 | 1.31 | 1.37 | 1.37 | 1.45 |
| 28 | 1.05 | 1.46 | 1.42 | 1.32 | 1.30 | 1.29 | 1.34 | 1.46 |
| 29 | 1.00 | 1.42 | 1.41 | 1.31 | 1.31 | 1.33 | 1.32 | 1.44 |
| 30 | 1.02 | 1.41 | 1.44 | 1.34 | 1.31 | 1.28 | 1.35 | 1.47 |
| 31 | 1.00 | 1.44 | 1.43 | 1.38 | 1.30 | 1.36 | 1.34 | 1.45 |
| 32 | 1.11 | 1.46 | 1.43 | 1.37 | 1.30 | 1.31 | 1.34 | 1.45 |
| 33 | 1.00 | 1.46 | 1.42 | 1.35 | 1.27 | 1.34 | 1.33 | 1.46 |
| 34 | 1.03 | 1.47 | 1.24 | 1.34 | 1.33 | 1.32 | 1.34 | 1.47 |
| 35 | 1.00 | 1.45 | 1.56 | 1.37 | 1.31 | 1.34 | 1.37 | 1.46 |
| 36 | 1.04 | 1.44 | 1.46 | 1.36 | 1.29 | 1.32 | 1.36 | 1.47 |
| 37 | 1.01 | 1.47 | 1.46 | 1.35 | 1.32 | 1.34 | 1.35 | 1.46 |

TABLE 28. This table is based at each k on a random sample of 10 blocks of length k on two symbols. M is the maximum preperiod from the sample. The maps are the same as used in Table 27 (Remark: For $x \in P_k(S_2)$ with $k = q2^j$ with q odd, the point $A^{j+1}(x)$ must be periodic under f , so $M \leq j + 1$.)

| k | p | μ | k | p | μ | k | p | μ | k | p | μ |
|-----|-----|-------|-----|------|-------|-----|--------|-------|-----|----------|-------|
| 1 | 1 | 10 | 12 | 1 | 10 | 20 | 3 | 1 | 30 | 31 | 1 |
| 2 | 1 | 10 | 13 | 47 | 6 | | 2790 | 9 | | 82531 | 9 |
| 3 | 1 | 10 | | 52 | 4 | 21 | 573 | 10 | 31 | 57747 | 10 |
| 4 | 1 | 10 | 14 | 1 | 1 | 22 | 11 | 3 | 32 | 85 | 2 |
| 5 | 1 | 1 | | 5 | 3 | | 519 | 3 | | 91 | 2 |
| | | 3 | | 13 | 3 | | 9658 | 4 | | 234649 | 6 |
| 6 | 1 | 5 | | 47 | 2 | 23 | 1499 | 8 | 33 | 3452570 | 6 |
| | | 3 | | 49 | 1 | | 9384 | 2 | | 10357710 | 4 |
| 7 | 1 | 3 | 15 | 31 | 7 | 24 | 1 | 1 | 34 | 1717 | 7 |
| | | 5 | | 145 | 3 | | 35160 | 9 | | 10574 | 1 |
| 8 | 1 | 5 | 16 | 29 | 1 | 25 | 20475 | 10 | | 999056 | 2 |
| | | 13 | | 85 | 9 | 26 | 441 | 1 | 35 | 572068 | 1 |
| 9 | 1 | 1 | 17 | 101 | 3 | | 9401 | 9 | | 2860340 | 1 |
| | | 9 | | 399 | 7 | 27 | 4543 | 5 | | 3262280 | 8 |
| 10 | 1 | 1 | 18 | 1 | 1 | | 19710 | 1 | 36 | 56 | 2 |
| | | 3 | | 455 | 9 | | 113643 | 4 | | 4095 | 1 |
| | | 5 | 19 | 1 | 1 | 28 | 5 | 1 | | 729537 | 3 |
| | | 11 | | 401 | 4 | | 1260 | 9 | | 908910 | 1 |
| 11 | 11 | 6 | | 2755 | 3 | 29 | 277298 | 10 | | 2188611 | 3 |
| | 143 | 4 | | 7125 | 2 | | | | 37 | 5881335 | 2 |
| | | | | | | | | | | 12081277 | 8 |

TABLE 29. This table is based at each k on a random sample by FProbPeriod of 10 blocks of length k on symbols 0, 1. It is a table of the resulting periods p with multiplicities μ for the nonclosing map $C = B \circ B_{rev}$ where $B = x_0 + x_1x_2$ and $B_{rev} = x_0x_1 + x_2$. Here, a sampled block $x[0, k - 1]$ determines a point x of period k , and the period p is by definition the eventual period of x under iteration by C . The multiplicity μ for a given period is the number of samples for which x has that period.

| k | p | μ | k | p | μ | k | p | μ | k | p | μ | k | p | μ |
|-----|-----|-------|-----|------|-------|-----|--------|-------|-----|----------|-------|-----|----------|-------|
| 1 | 1 | 1 | 11 | 11 | 7 | 19 | 2755 | 3 | 27 | 113643 | 6 | 34 | 1717 | 10 |
| 2 | 1 | 1 | | 143 | 3 | | 7125 | 5 | | 122661 | 4 | 35 | 3262280 | 8 |
| 3 | 1 | 7 | 12 | 1 | 9 | | 7619 | 2 | 28 | 35 | 2 | | 6886355 | 2 |
| | 3 | 3 | | 6 | 1 | 20 | 1395 | 9 | | 180 | 5 | 36 | 56 | 2 |
| 4 | 1 | 6 | 13 | 13 | 2 | | 5780 | 1 | | 1260 | 2 | | 504 | 2 |
| | 2 | 4 | | 52 | 5 | 21 | 4011 | 10 | | 26124 | 1 | | 729537 | 3 |
| 5 | 1 | 2 | | 611 | 3 | 22 | 11 | 1 | 29 | 277298 | 10 | | 2188611 | 3 |
| | 5 | 3 | 14 | 7 | 2 | | 878 | 5 | 30 | 5205 | 1 | 37 | 3768043 | 1 |
| | 15 | 5 | | 35 | 2 | | 5709 | 2 | | 137190 | 1 | | 5881335 | 2 |
| 6 | 1 | 7 | | 91 | 6 | | 9658 | 2 | | 1237965 | 8 | | 12081277 | 7 |
| | 3 | 3 | 15 | 87 | 2 | 23 | 9384 | 2 | 31 | 457777 | 1 | | | |
| 7 | 7 | 2 | | 465 | 8 | | 34477 | 8 | | 1790157 | 9 | | | |
| | 35 | 8 | 16 | 680 | 7 | 24 | 1 | 2 | 32 | 680 | 1 | | | |
| 8 | 1 | 1 | | 728 | 3 | | 11720 | 5 | | 728 | 3 | | | |
| | 4 | 2 | 17 | 1717 | 5 | | 35160 | 3 | | 267824 | 2 | | | |
| | 52 | 7 | | 6783 | 5 | 25 | 4095 | 8 | | 3754384 | 4 | | | |
| 9 | 1 | 1 | 18 | 9 | 1 | | 20475 | 2 | 33 | 3452570 | 5 | | | |
| | 9 | 9 | | 56 | 1 | 26 | 52 | 2 | | 10357710 | 5 | | | |
| 10 | 15 | 7 | | 4095 | 8 | | 122213 | 8 | | | | | | |
| | 55 | 3 | | | | | | | | | | | | |

TABLE 30. This table is constructed just as Table 29 was, except for the following: the data is for the map D which is the map C composed with $(S_2)^{-2}$, i.e., D is the composition of $x_0 + x_1x_2$ with $x_{-2}x_{-1} + x_0$.

| k | p | μ_{30} | μ_{10} | k | p | μ_{30} | μ_{10} | k | p | μ_{30} | μ_{10} |
|-----|-----|------------|------------|-----|--------|------------|------------|-----|---------|------------|------------|
| 1 | 1 | 30 | 10 | 17 | 255 | 30 | 10 | 36 | 252 | 30 | 10 |
| 2 | 1 | 30 | 10 | 18 | 126 | 30 | 10 | 37 | 3233097 | 30 | 10 |
| 3 | 1 | 7 | 3 | 19 | 9709 | 30 | 10 | 38 | 19418 | 30 | 10 |
| | 3 | 23 | 7 | 20 | 30 | 1 | 1 | 39 | 4095 | 30 | 10 |
| 4 | 1 | 30 | 10 | | 60 | 29 | 9 | 40 | 120 | 30 | 10 |
| 5 | 15 | 30 | 10 | 21 | 63 | 30 | 10 | 41 | 41943 | 30 | 10 |
| 6 | 1 | 3 | 0 | 22 | 682 | 30 | 10 | 42 | 126 | 30 | 10 |
| | 3 | 9 | 3 | 23 | 2047 | 30 | 10 | 43 | 5461 | 30 | 10 |
| | 6 | 18 | 7 | 24 | 24 | 30 | 10 | 44 | 1364 | 30 | 10 |
| 7 | 7 | 30 | 1 | 25 | 25575 | 30 | 10 | 45 | 4095 | 30 | 10 |
| 8 | 1 | 30 | 9 | 26 | 1638 | 30 | 10 | 46 | 4094 | 30 | 10 |
| 9 | 63 | 30 | 10 | 27 | 13797 | 30 | 10 | 47 | 8388607 | 30 | 10 |
| 10 | 15 | 1 | 10 | 28 | 28 | 30 | 10 | 48 | 48 | 30 | 10 |
| | 30 | 29 | 10 | 29 | 475107 | 30 | 10 | 49 | 2097151 | 30 | 10 |
| 11 | 341 | 30 | 10 | 30 | 30 | 30 | 10 | | | | |
| 12 | 12 | 30 | 10 | 31 | 31 | 30 | 10 | | | | |
| 13 | 819 | 30 | 10 | 32 | 1 | 30 | 10 | | | | |
| 14 | 14 | 30 | 10 | 33 | 1023 | 30 | 10 | | | | |
| 15 | 15 | 30 | 10 | 34 | 510 | 30 | 10 | | | | |
| 16 | 1 | 30 | 10 | 35 | 4095 | 30 | 10 | | | | |

TABLE 31. Table for $A = x_0 + x_1$ constructed as for Table 29, for sample sizes 10 and 30 for FProbPeriod. Both sample sizes work well for this linear map.

| k | Pre. | k | Pre. | k | Pre. | k | Pre. | k | Pre. | k | Pre. |
|-----|------|------|-------|-------|------|--------|--------|----|-------|----|--------|
| 18 | 0 | 21 | 216 | 24 | 394 | 27 | 2669 | 30 | 7988 | 33 | 100664 |
| | 45 | | 232 | | 1032 | | 3329 | | 8537 | | 108279 |
| | 170 | | 254 | | 1627 | | 5641 | | 10545 | | 146726 |
| | 226 | | 578 | | 1834 | | 9244 | | 12479 | | 149017 |
| | 293 | | 595 | | 2880 | | 9697 | | 13204 | | 157529 |
| | 362 | | 756 | | 3145 | | 11036 | | 13676 | | 161009 |
| | 506 | | 1174 | | 3396 | | 11583 | | 16210 | | 188071 |
| | 556 | | 1313 | | 3905 | | 13921 | | 16315 | | 196758 |
| | 751 | | 3058 | | 5215 | | 14745 | | 24373 | | 240207 |
| | 102 | | 3186 | | 5517 | | 23357 | | 31106 | | 270439 |
| 19 | 0 | 22 | 73 | 25 | 57 | 28 | 6000 | 31 | 4184 | 34 | 74577 |
| | 9 | | 350 | | 404 | | 7790 | | 11132 | | 80674 |
| | 145 | | 381 | | 463 | | 14999 | | 16211 | | 161429 |
| | 201 | | 522 | | 750 | | 18569 | | 30108 | | 193935 |
| | 291 | | 587 | | 1361 | | 23067 | | 30661 | | 209842 |
| | 301 | | 1163 | | 1547 | | 25108 | | 32789 | | 283852 |
| | 490 | | 1480 | | 1671 | | 28943 | | 39001 | | 360139 |
| | 547 | | 1788 | | 2491 | | 31291 | | 57399 | | 400913 |
| | 658 | | 3131 | | 9415 | | 37745 | | 70339 | | 452695 |
| | 105 | | 3247 | | 9531 | | 47285 | | 99228 | | 521740 |
| | 20 | | 0 | | 23 | | 0 | | 26 | | 428 |
| 44 | | 0 | 7866 | 9704 | | 14007 | 51383 | | | | |
| 181 | | 671 | 9538 | 10239 | | 19792 | 71921 | | | | |
| 230 | | 2543 | 11744 | 10783 | | 48308 | 82039 | | | | |
| 257 | | 2791 | 12056 | 11194 | | 49195 | 122647 | | | | |
| 290 | | 2835 | 13070 | 13472 | | 65168 | 261207 | | | | |
| 445 | | 2900 | 15762 | 19429 | | 65824 | 416233 | | | | |
| 556 | | 3123 | 16164 | 22208 | | 72902 | 492662 | | | | |
| 622 | | 6492 | 16999 | 31392 | | 126309 | 529999 | | | | |
| 755 | | 7854 | 17346 | 31856 | | 184781 | 555899 | | | | |

TABLE 32. This table is based at each k on a random sample of 10 blocks of length k on symbols 0, 1. It is a table of the preperiods (Pre.) seen by FProbPeriod in a sample for the map $B = x_0 + x_1x_2$.

| k | Pre. | k | Pre. | k | Pre. | k | Pre. | k | Pre. | k | Pre. |
|----|------|----|------|----|-------|----|-------|----|-------|----|--------|
| 18 | 0 | 21 | 134 | 24 | 316 | 27 | 2016 | 30 | 1862 | 33 | 1790 |
| | 15 | | 159 | | 969 | | 2046 | | 2020 | | 6742 |
| | 17 | | 286 | | 1012 | | 2543 | | 2159 | | 8823 |
| | 26 | | 305 | | 1027 | | 3793 | | 6736 | | 19037 |
| | 53 | | 551 | | 1541 | | 5981 | | 15523 | | 19158 |
| | 98 | | 600 | | 1954 | | 10577 | | 16289 | | 19234 |
| | 113 | | 696 | | 2800 | | 13024 | | 29590 | | 33862 |
| | 177 | | 748 | | 3054 | | 14611 | | 32253 | | 56053 |
| | 184 | | 1379 | | 3371 | | 15185 | | 56898 | | 105560 |
| | 198 | | 1550 | | 5430 | | 17428 | | 59246 | | 119315 |
| 19 | 15 | 22 | 7 | 25 | 1215 | 28 | 982 | 31 | 2176 | 34 | 28492 |
| | 35 | | 14 | | 1379 | | 5042 | | 3552 | | 176703 |
| | 48 | | 202 | | 3305 | | 5438 | | 3716 | | 184042 |
| | 49 | | 241 | | 3429 | | 5783 | | 16352 | | 203938 |
| | 108 | | 389 | | 3486 | | 5890 | | 30302 | | 204854 |
| | 119 | | 436 | | 3673 | | 8235 | | 53475 | | 261574 |
| | 211 | | 481 | | 3865 | | 9723 | | 55921 | | 282167 |
| | 264 | | 1003 | | 4716 | | 10118 | | 58664 | | 295105 |
| | 271 | | 1119 | | 4719 | | 15474 | | 60608 | | 374994 |
| | 275 | | 1477 | | 6056 | | 19004 | | 65768 | | 384473 |
| 20 | 7 | 23 | 177 | 26 | 289 | 29 | 7160 | 32 | 2686 | 35 | 15491 |
| | 20 | | 198 | | 327 | | 8519 | | 4683 | | 68409 |
| | 136 | | 341 | | 1127 | | 9150 | | 9120 | | 76254 |
| | 138 | | 366 | | 1301 | | 16361 | | 9808 | | 134451 |
| | 273 | | 793 | | 4720 | | 16679 | | 18985 | | 168960 |
| | 280 | | 1036 | | 5146 | | 21158 | | 19562 | | 205781 |
| | 422 | | 1043 | | 5817 | | 22279 | | 32081 | | 223889 |
| | 474 | | 1402 | | 8610 | | 24581 | | 76390 | | 247831 |
| | 478 | | 2389 | | 11410 | | 25450 | | 89515 | | 351861 |
| | 614 | | 2613 | | 12155 | | 25558 | | 98677 | | 385646 |

TABLE 33. This table is based at each k on a random sample of 10 blocks of length k on symbols 0, 1. It is a table of the preperiods (Pre.) seen by FProbPeriod in a sample for the nonclosing map $C = B \circ B_{rev}$, where $B = x_0 + x_1x_2$ and $B_{rev} = x_0x_1 + x_2$.

REFERENCES

- [Boyle-Kitchens1999] M. Boyle and B. Kitchens, *Periodic points for onto cellular automata*, Indag. Mathem., N.S., **10** (1999), no. 4, 483-493.
- [Hedlund-Appel-Welch1963] G. A. Hedlund, K. I. Appel and L. R. Welch, *All onto functions of span less than or equal to five*, IDA-CRD Working Paper (July, 1963), 73 pages.

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