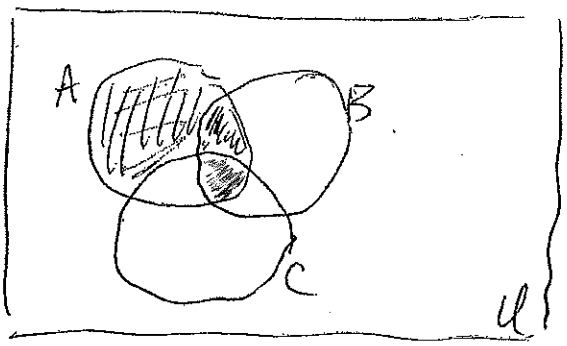
MATH 131
EXAM 3
FALL 2015
ANSWERS

1(a)



$A \cap (B \cup C')$ is shaded.

1(b) Sample space $S = \{(12), (13), (14), (23), (24), (34)\}$

The event "both even" is $\{(24)\} := E$

$\text{Prob}(E) = \frac{\#E}{\#S} = \boxed{\frac{1}{6}}$.

2(a) $(1+2+3+4+5+6)/6 = \boxed{3.5}$

(b) $(2)(3.5) = \boxed{7}$

(c) There are 4 ways to total 5: (14), (23), (32), (41).
So this probability is $\frac{4}{36} = \boxed{\frac{1}{9}}$

(d) $A \cap B$ is the event $\{(32)\}$

$P(A \cap B) = 1/36$

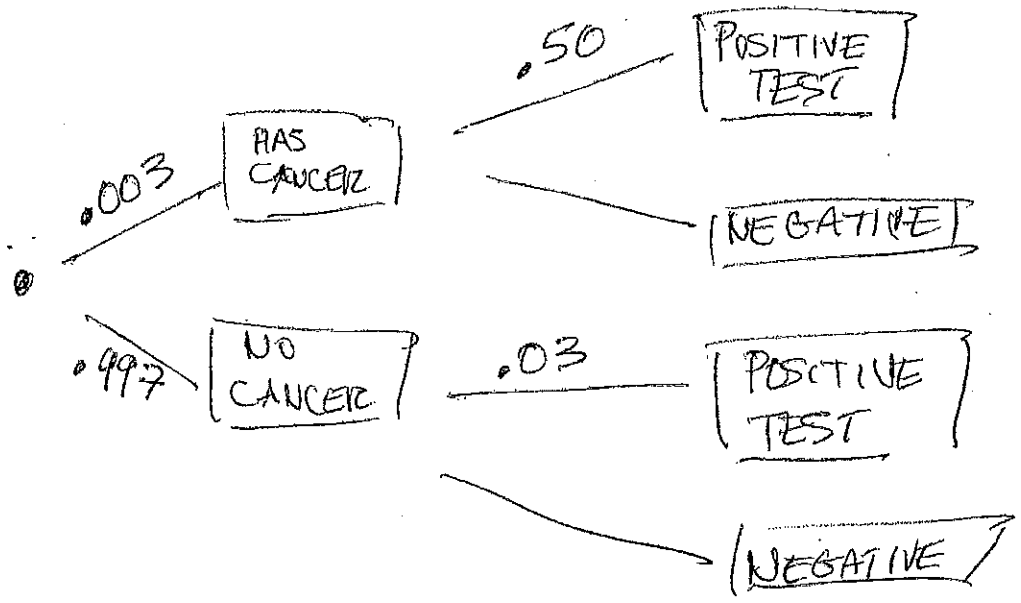
$(P(A))(P(B)) = (1/6)(1/9) = 1/54 \neq \frac{1}{36}$.

So, A and B are **not independent**.

3(a) $(3 - (1.96)(10), 3 + (1.96)(10))$

(b) FALSE (c) FALSE (d) FALSE

4.



Prob (cancer | positive test)

$$= \boxed{\frac{(.003)(.50)}{(.003)(.50)+(.997)(.03)}} \quad \left(\approx \frac{1}{21}\right)$$

5. (b) $\text{Prob}(X > t) = 1 - \text{Prob}(X \le t)$

$$= 1 - \int_{x=.1}^{t} 4x^{-5}\,dx = 1 - \left[-x^{-4}\right]_{x=.1}^{t}$$

$$= 1 - \left[\left(-\frac{1}{t^4}\right) - (-1)\right] = 1 - \left[\frac{-1}{t^4} + 1\right]$$

$$= \boxed{1/t^4} \qquad \boxed{\text{Prob}(X > 2) = 1/16}$$

(a) $$E(X) = \int_{x=1}^{\infty} (x)(4x^{-5})\,dx = \int_{1}^{\infty} 4x^{-4}\,dx = 4\left[\frac{-x^{-3}}{3}\right]_{1}^{\infty}$$

$$= \boxed{4/3}$$

6(a) $\text{st.dev}(S) = \sqrt{9}\,(\text{st.dev. of } \bar{X}_i) = \sqrt{9}\sqrt{100} = \boxed{30}$

$$\text{st.dev}\left(\bar{X}\right) = \frac{\sqrt{100}}{\sqrt{9}} = \frac{10}{\sqrt{9}} = \boxed{\frac{10}{3}}$$

(b)

$$\left(\bar{x} - (1.96)\frac{s}{\sqrt{n}},\ \bar{x} + (1.96)\frac{s}{\sqrt{n}}\right)$$

$$= \left(20 - \frac{(1.96)(5)}{\sqrt{70}},\ 20 + \frac{(1.96)(5)}{\sqrt{70}}\right)$$

7(a) $e^{-a(1/a)} = \boxed{\frac{1}{e}}$

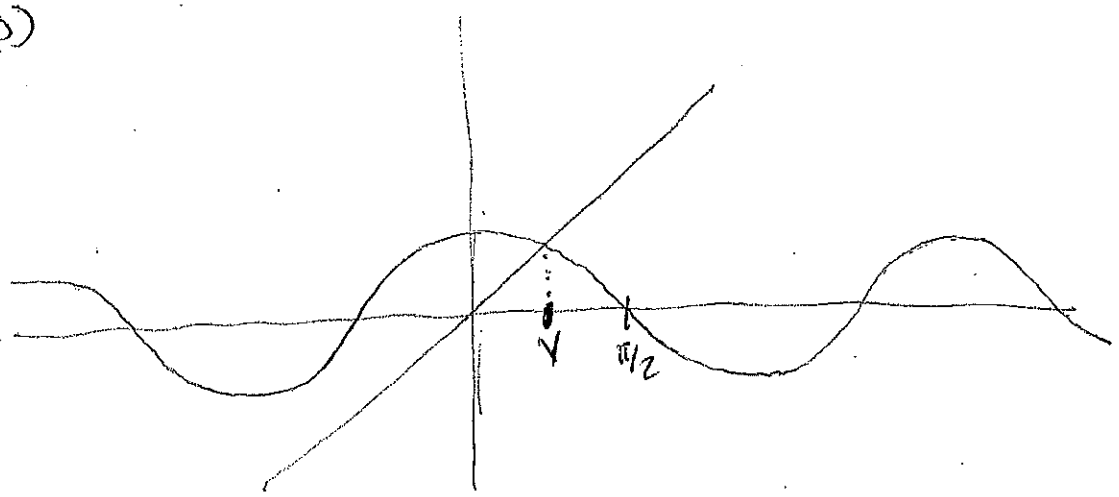
(b) $\boxed{\frac{1}{e}}$

8(a) $a_1 = \pi/2$

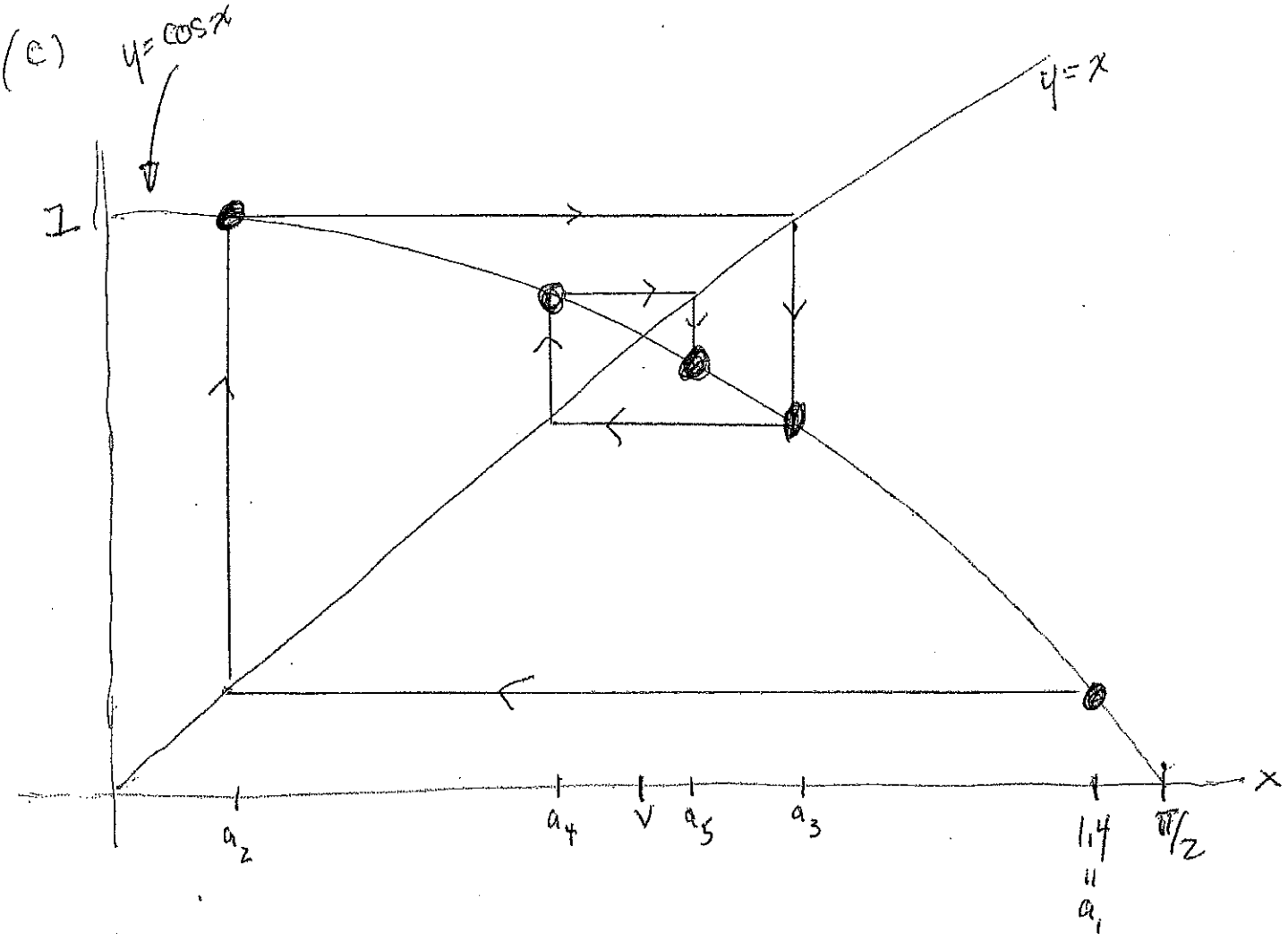
$a_2 = \cos(\pi/2) = 0$

$a_3 = \cos(0) = 1$

(b)



8(c)



(d) The equilibrium is stable.