Exam 1 – MATH 151 – FALL 2015 – Solutions

(1) Avg. value $= \frac{1}{\pi - 0}\int_0^{\pi} \sin x\,dx = \frac{1}{\pi}\left[-\cos x\right]_{x=0}^{\pi} = \frac{1}{\pi}\left[(-\cos\pi) - (-\cos 0)\right]$

$= \frac{1}{\pi}\left[1 - (-1)\right] = \boxed{\frac{2}{\pi}}$

(2) $\int_{x=1}^{3} \underbrace{x}_{u'} \underbrace{\ln x}_{v}\,dx = \left[\underbrace{\frac{1}{2}x^2}_{u}\underbrace{\ln x}_{v}\right]_{x=1}^{3} - \int_{x=1}^{3} \underbrace{\left(\frac{1}{2}x^2\right)}_{u}\underbrace{\left(\frac{1}{x}\right)}_{v'}dx$

$= \left(\frac{9}{2}\ln 3\right) - (0) - \int_1^3 \frac{1}{2}x\,dx$

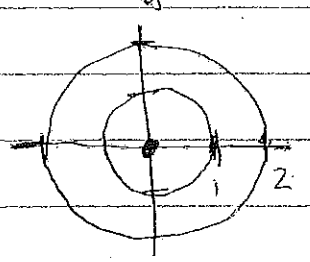
$= \frac{9\ln 3}{2} - \left[\frac{1}{4}x^2\right]_{x=1}^{3} = \frac{9\ln 3}{2} - \left[\left(\frac{9}{4}\right) - \left(\frac{1}{4}\right)\right]$

$= \boxed{\frac{9\ln 3}{2} - 2}$

(3) $\int_{x=1}^{\infty} \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty}\int_{x=1}^{b} x^{-1/2}\,dx = \lim_{b\to\infty}\left[2x^{1/2}\right]_{x=1}^{b}$

$= \lim_{b\to\infty}\left[\left(2b^{1/2}\right) - (2)\right] = \lim_{b\to\infty}\left[2\sqrt{b} - 2\right] = \boxed{\infty}$

(4)



level curve at
$z = 0$ is one point, $(0,0)$
$z = 1$ is circle of radius 1 around $(0,0)$
$z = 4$ " " " " 2 " "

(5) Let $f(x,y) = \sqrt{x^2+y^2}$

$f_x = x/\sqrt{x^2+y^2}$

$f_y = y/\sqrt{x^2+y^2}$

$\sqrt{(8.04)^2+(5.98)^2} \approx f(8,6) + f_x(8,6)\Delta x + f_y(8,6)\Delta y$

$= 10 + \frac{8}{10}(.04) + \frac{6}{10}(-.02) = 10 + .032 - .012 = \boxed{10.02}$

(6) (a) $f = x^2 + y^2 + xy - 6x - 3$

$f_x = 2x + y - 6$

$f_y = 2y + x$

solve for $f_x = 0 = f_y$:

$0 = f_y = 2y + x \Rightarrow x = -2y$. Then

$0 = f_x = 2x + y - 6 = 2(-2y) + y - 6$

$0 = -3y - 6 \Rightarrow y = -2$ Then

$x = -2y = -2(-2) = 4$

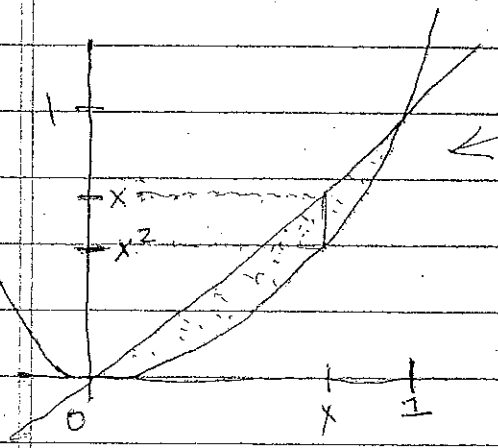
The only critical point is $(4, -2)$

(b) $f_{xx} = 2$, $f_{xy} = f_{yx} = 1$, $f_{yy} = 2$.

$D = (f_{xx})(f_{yy}) - (f_{xy})^2 = (2)(2) - (1)^2 = 3$

Because $D > 0$ and $f_{xx} > 0$, f has a local minimum at $(4, -2)$

(7)



$$R \quad \int_{x=0}^{1} \int_{y=x^2}^{x} x^2 y \, dy \, dx$$

$$= \int_{x=0}^{1} \left[x^2 \left(\frac{1}{2} y^2 \right) \right]_{y=x^2}^{x} dx$$

$$= \int_{x=0}^{1} \frac{1}{2} x^4 - \frac{1}{2} x^6 \, dx = \left[\frac{1}{10} x^5 - \frac{1}{14} x^7 \right]_{x=0}^{1}$$

$$= \frac{1}{10} - \frac{1}{14} = \frac{1}{35}$$

(8)

(a) $k = 1$ $m = 2$ $p = 4$

(b) (i) $n = 160{,}000$

(ii) $n = 160$

(c) $x_0, x_1, x_2, x_3, x_4 = -2, -1, 0, 1, 2$