Math 131 – Fall 2015 – Boyle –Exam 3

- No calculators or electronic devices allowed.
- YOU DO NOT HAVE TO SIMPLIFY ARITHMETIC EXPRESSIONS.
- One answer sheet for each of the eight questions.
- Draw a box around a final numerical answer.

1.(a) (4 points) Draw a Venn diagram for sets A, B, C contained in a universal set U. Use shading to represent the set $A \cap (B \cup C')$.
(b)(10 pts.) Four slips of paper are labeled 1,2,3,4 (each number is on exactly one slip of paper) and put in a box. Two slips are taken out simultaneously.
(i) Define a sample space to represent the outcome of this experiment.
(ii) What is the probability both slips drawn are labeled by even numbers?

2. Two dice are rolled. The 36 outcomes are equally likely.
(a)(3 pts.) What's the expected value of the number rolled on the first die?
(b)(3 pts.) What's the expected value of the total of the two numbers rolled?
(c)(4 pts.) What is the probability that the total rolled is 5?
(d)(4 pts.) Let A be the event that the first die rolls 3. Let B be the event that the total of the two is 5. Are these events independent? (Briefly justify.)

3. (a)(4 pts.) Suppose X is a random variable with distribution $\mathcal{N}(\mu, \sigma)$ where $\mu = 3$ and $\sigma = 10$. Find an interval (a, b) such that $a < X < b$ with probability approximately .95 .

For (b),(c),(d) answer True or False. 3 pts. each.

(b) If X is a random variable and Y is its standardization, then the standardization of $X + 2$ is $Y + 2$.
(c) If two events are mutually exclusive, then the events are independent.
(d) If a random variable X has uniform distribution on the interval $[0, 3]$, then the event $X > 2$ has probability 1.

4. (12 points) The probability of colorectal cancer can be given as 0.3%. If a person has this cancer, the probability a hemoccult test is positive is 50%. If a person does not have this cancer, the probability of a positive test is 3%. What is the probability that a person who tests positive has this cancer?

5. (12 points) Suppose f is a probability density function for a random variable X, with $f(x) = 4x^{-5}$ if $x \geq 1$ and $f(x) = 0$ if $x < 1$.
(a) What is the expected value of X?
(b) What is the probability that $X > 2$?

6. (a) (6 points) Suppose $X_1, \ldots, X_9$ are i.i.d. random variables, each with mean 2 and variance 100.
(i) Let $S = X_1 + \cdots + X_9$. What is the standard deviation of S?
(ii) Let $\overline{X} = (X_1 + \cdots + X_9)/9$. What is the standard deviation of $\overline{X}$?

(b) (6 points) Suppose a random sample of 70 people from a specified obese population participate in a certain dieting program, and their average weight loss is 20 pounds, with sample standard deviation 5 pounds. Give a 95% confidence interval (in pounds) for the average weight loss expected for persons from this population participating in the program.

7. The number of days between major earthquakes in the north-south seismic belt of China is modeled as a random variable X with an exponential distribution; for $x \geq 1$, the probability density function is $f(x) = ae^{-ax}$, with $a = 1/609.5$.
(a) (6 points) What is the probability that the time between a major earthquake and the next one is greater than 609.5 days?
(b) (6 points) Suppose you have already waited 609.5 days without having an earthquake. What is the probability that you have to wait at least another 609.5 days for the next earthquake?

8. Given a number a_1, define a sequence $a_1, a_2, a_3, \ldots$ by the recursive rule $a_n = f(a_{n-1})$ if $n > 1$, where $f(x) = \cos x$.
(a) (2 points) Given $a_1 = \pi/2$, compute a_2 and a_3.
(b) (3 points) Draw appropriate graphs to find an equilibrium value (call it v) for this iterated function system. Mark the location of v on the horizontal axis in your graphs picture. You don't need to provide a numerical estimate for v.
(c) (5 points) Given $a_1 = 1.4$, draw a cobweb diagram marking the points $(a_n, f(a_n))$ on the graph of $f(x) = \cos(x)$ for $n = 1, 2, 3, 4, 5$. Draw this diagram on the back of your answer sheet, using almost the entire width for the horizontal axis segment $[0, \pi/2]$. ($\pi/2$ is approximately 1.57)
(d) (3 points) Is your equilbrium stable or unstable?