Math 131 – Spring 2016 – Boyle –Exam 2

- NO CALCULATORS OR ELECTRONIC DEVICES ALLOWED.
- Use a separate answer sheet for each question; use the back side of an answer sheet if you need more space to answer a question.
- Give your pledge on page 1 only, covering the whole test.
- Draw a box around a final answer to a problem.

1. (a) (5 pts.) From a class of ten students, a team of 3 students will be chosen. What is the number of possible teams?
(b) (5pts.) Ten people remain in competition at an Olympic event. How many different ways remain to award the gold medal, the silver model and the bronze medal to three of these ten people?

2. Suppose a function f of two variables is defined for positive numbers x and y by the rule $f(x,y) = xy + \frac{1}{x} + \frac{1}{y}$.
(a) (5 pts.) Find every critical point of f.
(b) (10 pts.) At each critical point, determine whether f has a local maximum, local minimum or saddle point. Show work.

3. (Chemotaxis) Suppose a bacterium finds food by moving toward the highest concentration of glucose. Suppose it starts at the point $(-2,1)$ in the xy-plane, and the concentration of glucose, in certain units, is

$$f(x,y) = \frac{1}{3 + x^2 + y^2}$$

(a) (3 pts.) Draw the level curve of f which contains $(-2,1)$.
(b) (7 pts.) Find the direction in which the bacterium will move initially.
(c) (2 pts.) Draw an arrow in this direction which starts at $(-2,1)$.

4. (10 pts) Suppose $z = x^2y^3$, with x and y functions of t such that when $t = 0$ we have $x = 1$, $y = -1$, $dx/dt = 2$ and $dy/dt = -2$.
What is dz/dt when $t = 0$?

5. (a) (8 pts.) Find the linear approximation (the linearization) of the function $f(x,y) = \ln(x - 3y)$ at $(7,2)$.
(b) (5 pts.) Use it to approximate $f(6.9, 2.06)$.

6. (a) (5 pts.) Compute the inverse of the matrix $A = \left(\begin{smallmatrix} 3 & 1 \\ 11 & 7 \end{smallmatrix} \right)$.
(b) (9 pts.) For each of the following, answer True or False. (No justification required)
(i) If an invertible matrix B has eigenvalues 3 and 4, then B^{-1} has eigenvalues -3 and -4.
(ii) If a matrix B has eigenvalues 3 and 4, then B^{10} has eigenvalues 30 and 40.
(iii) If A, B, C are 5×5 matrices such that A is invertible and $AB = AC$, then $B = C$.

7. In a certain population, a gene can appear as two possible alleles, A and B. The process of mutation is modeled by the recursion $p_A(t+1) = .3p_A(t) + .4p_B(t)$ and $p_B(t+1) = .7p_A(t) + .6p_B(t)$, where $p_A(t)$ and $p_B(t)$ are the proportion of the population at time t with the corresponding allele. Define

$$p(t) = \begin{pmatrix} p_A(t) \\ p_B(t) \end{pmatrix} .$$

(a) (4 pts.) Find a matrix M such that $p(t+1) = Mp(t)$.
(b) (10 pts.) Suppose $p_A(0) = 1$ and $p_B(0) = 0$. After a long time, approximately what fraction of the population has allele A?

8. Let $x_1(t)$ and $x_2(t)$ be the sizes of two populations at time t. Suppose the evolution is modeled by the linear differential equation $dx/dt = Ax$, where A is a 2×2 matrix and

$$x(t) = x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \text{and} \quad \frac{dx}{dt} = \begin{pmatrix} dx_1/dt \\ dx_2/dt \end{pmatrix} \quad \text{and} \quad x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(i) (6 pts.) For $A = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/3 \end{pmatrix}$, which of $(a) - (e)$ below are true?
(ii) (6 pts.) For $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$, which of $(a) - (e)$ below are true?
(a) As $t \to \infty$, the distance from $x(t)$ to the origin goes to ∞.
(b) As $t \to \infty$, the distance from $x(t)$ to the origin goes to zero.
(c) The solution curve is a spiral.
(d) The solution curve is a closed curve.
(e) The equilibrium point is a saddle.