EXAM 2 – MATH 131 – SPRING 2016 – BOYLE

1(a) $\binom{10}{3} = \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1} = 5\cdot 3\cdot 8 = \boxed{120}$

(b) $10\cdot 9\cdot 8 = \boxed{720}$

2(a) $\frac{\partial f}{\partial x} = y - \frac{1}{x^2} = 0 \rightarrow y = \frac{1}{x^2}$

$\frac{\partial f}{\partial y} = x - \frac{1}{y^2} = 0$ $\rightarrow x - \frac{1}{(1/x^2)} = 0$

So, $x - x^4 = 0$, $x(1-x^3) = 0$

The positive soln is $x = 1$; then $y = \frac{1}{1^2} = 1$.

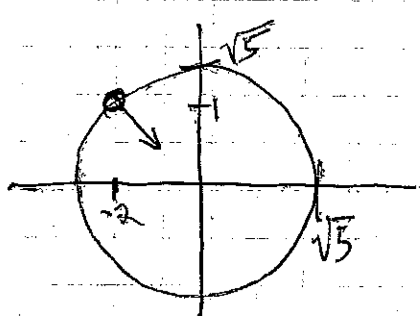
$\boxed{(1,1) \text{ is the only critical point}}$

(b) $\begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} = \begin{pmatrix} 2/x^3 & 1 \\ 1 & 2/y^3 \end{pmatrix} \xrightarrow{\text{at } (1,1)} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$\det\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = 3 > 0$ and $2+2 > 0$

$\boxed{f \text{ has a local min. at } (1,1)}$

3(a)



$f(-2,1) = 1/8$

$\frac{1}{8} = \frac{1}{3+x^2+y^2}$

$3 + x^2 + y^2 = 8$

$x^2 + y^2 = 5$

$\boxed{\text{level curve is a circle of radius } \sqrt{5}}$

(b) moves in direction of ∇f:

$\nabla f(-2,1) = \left(\frac{-2x}{(3+x^2+y^2)^2}, \frac{-2y}{(3+x^2+y^2)^2} \right)\Big|_{(-2,1)}$

$= \frac{+2}{(3+x^2+y^2)^2}(-x,-y)\Big|_{(-2,1)} = \boxed{\frac{1}{32}(2,-1)}$

(C) See picture.

4. Chain Rule:

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$\left.\frac{dz}{dt}\right|_{t=0} = \left[\left.\left(2xy^3\right)\right|_{(1,-1)} \cdot (2)\right] + \left[\left(3x^2y^2\right)_{(1,-1)} \cdot (-2)\right]$$

$$= (-2)(2) + (3)(-2)$$

$$= \boxed{-10}$$

5. The linearization of f at $(7,2)$ is the function

$$L(x,y) = f(7,2) + \left.\frac{\partial f}{\partial x}\right|_{(7,2)}(x-7) + \left.\frac{\partial f}{\partial y}\right|_{(7,2)}(y-2)$$

$$L = \ln(7-6) + \left.\left(\frac{1}{x-3y}\right)\right|_{(7,2)}(x-7) + \left.\left(\frac{-3}{x-3y}\right)\right|_{(7,2)}(y-2)$$

$$L = \underbrace{\ln(1)}_{0} + (1)(x-7) + (-3)(y-2)$$

$$\boxed{L = x - 3y - 1}$$

$$f(6.9, 2.06) \approx 6.9 - 3(2.06) - 1$$

$$= 6.9 - 6.18 - 1 = \boxed{-.28}$$

6 (a) $A = \begin{pmatrix} 3 & 1 \\ 11 & 7 \end{pmatrix}$, $A^{-1} = \frac{1}{\det A}\begin{pmatrix} 7 & -1 \\ -11 & 3 \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 7 & -1 \\ -11 & 3 \end{pmatrix}$

$$\boxed{A^{-1} = \begin{pmatrix} .7 & -.1 \\ -1.1 & .3 \end{pmatrix}}$$

(b) (i) FALSE (ii) FALSE (iii) TRUE

7 (a) $M = \begin{pmatrix} .3 & .4 \\ .7 & .6 \end{pmatrix}$

(b) The matrix M is primitive with eigen values $\lambda_1 = 1$ and $\lambda_2 = -.1$

Then a solution to $\begin{pmatrix} .3-1 & .4 \\ .7 & .6-1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} -.7 & .4 \\ .7 & -.4 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

is $\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$.

After a long time, approximately $\frac{4}{11}$ of the population has allele A.

8 (i) General solution is $x(t) = c_1 e^{\frac{1}{2}t} + c_2 e^{\frac{1}{3}t}$
Specific solution for $x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is $x(t) = e^{\frac{1}{2}t}$.
Either way (for $x(0) \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$), $\|x(t)\| \to \infty$ as $t \to \infty$.

(i) $\boxed{\text{(a) is true}}$

(ii) char polynomial of A is $p(t) = t^2 - 2t + 5$.
The roots are $\lambda = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)}}{2}$ (quadratic formula)

$\lambda = 1 \pm i\sqrt{5}$. Because RealPart$(\lambda) = 1 > 0$,

(ii) $\boxed{\text{(a) is true}}$. Because $\mathrm{Re}\,\lambda \neq 0$ and $\mathrm{Im}\,\lambda \neq 0$,

$\boxed{\text{(c) is true}}$