

Calculus II: Quiz 6 Solutions

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1. Evaluate the length of the cardioid

$$r = 1 + \cos \theta \text{ for } 0 \leq \theta \leq 2\pi.$$

(6 points)

Solution: Note the cardioid is symmetric about x axis. Thus,

$$\begin{aligned} L &= 2 \int_0^\pi \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta \\ &= 2 \int_0^\pi \sqrt{(1 + \cos \theta)^2 + (-\sin \theta)^2} d\theta \\ &= 2 \int_0^\pi \sqrt{1 + 2 \cos \theta + \sin^2 \theta + \cos^2 \theta} d\theta \\ &= 2 \int_0^\pi \sqrt{2 + 2 \cos \theta} d\theta \\ &= 2\sqrt{2} \int_0^\pi \sqrt{1 + \cos \theta} d\theta \\ &= 2\sqrt{2} \int_0^\pi \sqrt{2 \cos^2 \left(\frac{\theta}{2}\right)} d\theta \\ &= 4 \int_0^\pi \cos \left(\frac{\theta}{2}\right) d\theta \\ &= 8 \sin \left(\frac{\theta}{2}\right) \Big|_0^\pi \\ &= 8. \end{aligned}$$

□

2. Let $\{a_n\}_{n=m}^\infty$ be a sequence. Define what it means by a number L being the limit of $\{a_n\}_{n=m}^\infty$. (Give $\epsilon - \delta$ definition ONLY. No credit for anything else.)

(4 points)

Solution: A number L is the limit of $\{a_n\}_{n=m}^\infty$ if for every $\epsilon > 0$ there is an integer N such that

$$\text{if } n \geq N, \text{ then } |a_n - L| < \epsilon.$$

□