

Math 406, Spring 2021
HW03, due Wednesday, February 17¹

Reading: Read Chapters 3 and 4 of Crisman's text.

Graded Problems: Work the following problems for a grade. Turn them in on Gradescope.

Some problems are taken from the Online Version of Crisman's text:

<http://math.gordon.edu/ntic/>

Each problem is worth 20 points.

1. For each of the following linear Diophantine equations, either show that there are no solutions or write down the general solution.

- (a) $21x + 14y = 133$.
- (b) $132x + 28y = 39$.
- (c) $30x - 47y = 2$.
- (d) $10x + 25y = 100$.

2 (Problem 3.6.10). Find all simultaneous integer solutions to the system of linear equations:

$$\begin{aligned}x + y + z &= 100 \\x + 8y + 50z &= 156.\end{aligned}$$

(See the hint in the text if you get stuck.)

3. Suppose a, b and n and m are integers with n and m positive, and suppose that $a \equiv b \pmod{n}$. Show, by induction on m , that $a^m \equiv b^m \pmod{n}$ as well.

4. Recall that, if a and n are integers with $n > 0$, I write $\text{Mod}(a, n)$ for the remainder you get when you divide a into n .

- (a) Compute $\text{Mod}(3^k, 11)$ for the integers $k = 1, 2, 4, 8, 16$ and 32 . (Do this by hand. If you ever see a number bigger than 121 you're doing something wrong.)
- (b) Use what you did above to compute $\text{Mod}(3^{43}, 11)$. (**Hint:** Write 43 in binary.)

5. Can you have 50 coins, all of which are pennies, dimes or quarters, worth a total of \$3?

¹This version created Wednesday 24th March, 2021 at 19:37.